STEP, Collisions – Q5 (11/6/23)

Ball A of mass m, travelling with speed u on a smooth surface, collides directly with ball B of mass km, which is at rest. The coefficient of restitution between the two balls is e.

(i) With $k = 1$, what condition must apply to e for ball A to be at rest after the collision?

(ii) For a given k, what condition must apply to e for ball A to reverse its direction after the collision?

Solution

(i) Let v_A & v_B be the final speeds of A & B in the original direction of A.

By conservation of momentum, $mu = mv_A + mv_B$,

so that $u = v_A + v_B$

And by Newton's law of restitution, $v_B - v_A = eu$

Adding these eq'ns then gives $v_B = \frac{1}{2}u(e+1)$,

and $v_A = u - \frac{1}{2}u(e+1) = \frac{1}{2}u(1-e)$

Ball A will be at rest when $v_A = 0$; ie when $e = 1$.

(ii) The two eq'ns become $u = v_A + kv_B$ and $v_B - v_A = eu$

Adding these eq'ns then gives $v_B = \frac{u(e+1)}{(k+1)}$

and $v_A = \frac{u(e+1)}{(k+1)} - eu = \frac{u}{(k+1)}\big(e+1-e(k+1)\big) = \frac{u(1-ek)}{(k+1)}$

A will thus reverse its direction when $1 - ek < 0$; ie when $e > \frac{1}{k}$

[So reversal occurs more readily when e is larger, or when B has a larger mass. The initial speed of A has no effect.]