

STEP/Collisions – Q2 (11/6/23)

For two balls colliding directly on a smooth surface, show that kinetic energy is conserved when $e = 1$.

Solution

Let the two balls have masses m_A & m_B , initial speeds u_A & u_B and final speeds v_A & v_B (where the speeds are from left to right, and $u_A > 0$, with $u_A > u_B$).

Then, by conservation of momentum,

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B \quad (1)$$

and, by Newton's law of impact, $\frac{v_B - v_A}{u_A - u_B} = e = 1$ (2)

$$\text{Result to prove: } \frac{1}{2} m_A (v_A^2 - u_A^2) + \frac{1}{2} m_B (v_B^2 - u_B^2) = 0 \quad (3)$$

$$\text{From (1), } m_B (v_B - u_B) = m_A (u_A - v_A),$$

$$\text{and from (2), } (v_B + u_B) = (u_A + v_A).$$

Then, substituting into (3),

$$\begin{aligned} LHS &= \frac{1}{2} m_A (v_A - u_A)(v_A + u_A) + \frac{1}{2} m_B (v_B - u_B)(v_B + u_B) \\ &= \frac{1}{2} m_A (v_A - u_A)(v_A + u_A) + \frac{1}{2} m_A (u_A - v_A)(u_A + v_A) = 0, \end{aligned}$$

as required.