# STEP/Collisions, Q1 (11/6/23)

Two particles of the same mass are travelling directly towards each other, on a smooth surface. Particle A has a speed which is $\theta$ times that of particle B (where $\theta > 0$). The coefficient of restitution between A and B is $e$.

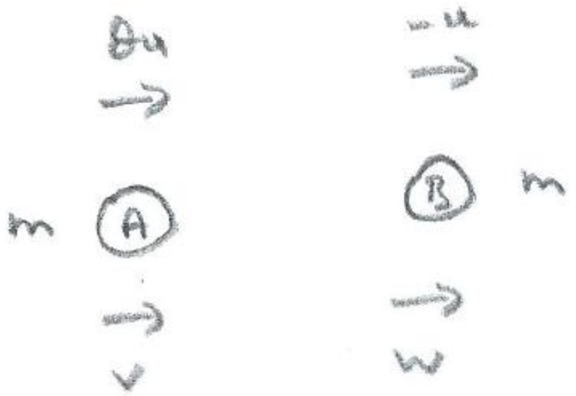
(i) Find the condition on $\theta$ that must apply in order for A to change direction on impact. Also give the condition on $e$.

(ii) Describe the motion of the particles after they have collided, in the case where $e = 0$.

(iii) Describe the motion of the particles after they have collided, in the case where $e = 1$.

(iv) In the case where $e = \frac{1}{3}$, describe the motion of the particles after they have collided, for the various possible values of $\theta$.

**Solution**




Conservation of momentum $\Rightarrow m(\theta u - u) = m(v + w)$, where $m$ is the mass of each particle, so that $(\theta - 1)u = v + w$ (1)

By Newton's Law of Restitution, $w - v = e(\theta u - (-u))$,

so that $eu(\theta + 1) = w - v$ (2)

Adding (1) & (2), $u(\theta - 1 + e\theta + e) = 2w$ (3)

Subtracting (2) from (1), $u(\theta - 1 - e\theta - e) = 2v$ (4)

(i) From (4), $v < 0 \Rightarrow \theta - 1 - e\theta - e < 0$ (as $u > 0$)

$\Rightarrow \theta(1 - e) < e + 1$

$\Rightarrow \theta < \frac{1+e}{1-e}$ , provided $e \neq 1$ (as $1 - e > 0$)

[If $e$ is close enough to 1, A will reverse its direction for any value of $\theta$ (the bigger $\theta$ is, the closer $e$ has to be to 1).]

Also $\theta - 1 - e\theta - e < 0 \Rightarrow \theta - 1 < e(\theta + 1) \Rightarrow e > \frac{\theta-1}{\theta+1}$

(ii) When $e = 0$, (4) & (3) $\Rightarrow v = w = \frac{(\theta-1)u}{2}$

Thus the particles coalesce, and travel in the original direction of the particle with the bigger speed.

(iii) When $e = 1$, (4) & (3) $\Rightarrow v = -u$ and $w = \theta u$

Thus both A and B have reversed their directions, and exchanged speeds.

(iv) When $e = \frac{1}{3}$, (4) & (3) $\Rightarrow v = \frac{u}{2}\ (\frac{2}{3}\theta - \frac{4}{3})$ and $w = \frac{u}{2}\ (\frac{4}{3}\theta - \frac{2}{3})$

$v < 0$ when $\frac{2}{3}\theta - \frac{4}{3} < 0$ ; ie $\theta < 2$

$w < 0$ when $\frac{4}{3}\theta - \frac{2}{3} < 0$ ; ie $\theta < \frac{1}{2}$

So, when $\theta < \frac{1}{2}$, both A and B move off to the left (in the diagram).

When $\theta = \frac{1}{2}$, A moves off to the left, and B comes to rest.

When $\frac{1}{2} < \theta < 2$, A moves off to the left and B moves off to the right.

When $\theta = 2$, A comes to rest, and B moves off to the right.

When $\theta > 2$, both A and B move off to the right.