STEP, Collisions, Q10 (11/6/23)

Two balls, A & B, collide directly on a smooth surface. Investigate the circumstances in which the loss of kinetic energy is maximised.

Solution

Let ball A have mass m, and speed u (say, from left to right),

whilst ball B has mass km, and speed λu (also from left to right, where λ could be negative).

Thus $\lambda < 1$, in order for a collision to take place.

Let the coefficient of restitution be e.

Suppose that the speeds after the collision are $v_A \, and \, v_B$.

Then, by conservation of momentum,

$mu + (km)(\lambda u) = mv_A + (km)v_B$,

and by NLR, $v_B - v_A = e(u - \lambda u)$,

Then, eliminating v_B, $u + k\lambda u = v_A + k(v_A + eu(1 - \lambda))$,

so that $v_A(1 + k) = u(1 + k\lambda - ke + ke\lambda)$

and hence $v_A = \frac{u(1+k\lambda-ke+ke\lambda)}{1+k}$

and $v_B = v_A + eu(1 - \lambda) = \frac{u(1+k\lambda-ke+ke\lambda+e(1+k)(1-\lambda))}{1+k}$

$= \frac{u(1+k\lambda-ke+ke\lambda+e-e\lambda+ek-ek\lambda)}{1+k} = \frac{u(1+k\lambda+e-e\lambda)}{1+k}$

Thus $v_A = \frac{u(1+k\lambda-k(1-\lambda)e)}{1+k}$ and $v_B = \frac{u(1+k\lambda+(1-\lambda)e)}{1+k}$

The loss of kinetic energy is then maximised when

${mv_A}^2 + km{v_B}^2$ is minimised;

ie when $(1 + k\lambda - k(1 - \lambda)e)^2 + k(1 + k\lambda + (1 - \lambda)e)^2$

is minimised.

This is a quadratic expression in e, with positive coefficient of e^2, and coefficient of e equal to:

$-2(1+k\lambda).\,k(1-\lambda)+2k(1+k\lambda)(1-\lambda)=0$,

which is therefore minimised when $e=0$.

[Note also that the loss of kinetic energy can be shown to be zero when $e=1$.]