STEP/Differentiation Q6 (15/6/23)

Find the turning points of $y = (x^2 - 4x + 3)^2$, and hence sketch the curve.

Solution

Method 1

As $x^2 - 4x + 3 = (x-1)(x-3)$,

$y = (x-1)^2(x-3)^2$

Then $\frac{dy}{dx} = 2(x-1)(x-3)^2 + (x-1)^2(2)(x-3)$

$= 2(x-1)(x-3)(x-3+x-1)$

$= 4(x-1)(x-3)(x-2)$

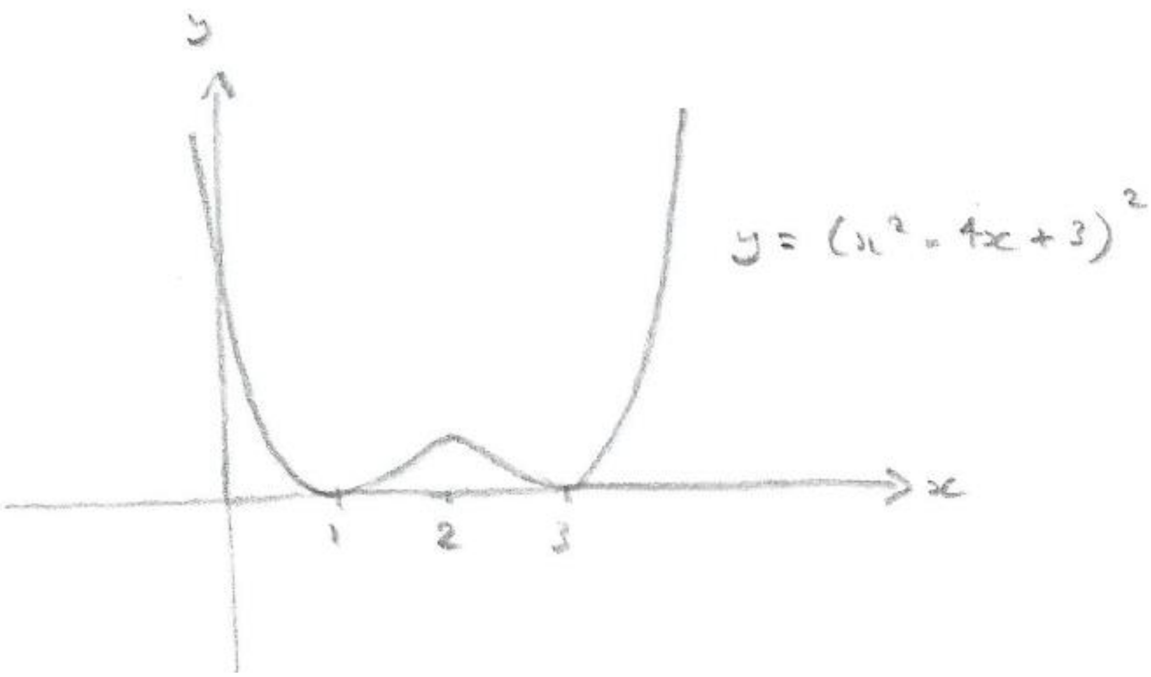
$\frac{dy}{dx} = 0$ when $x = 1, 2$ & 3

At $x = 1$, $\frac{dy}{dx}$ changes from –ve to +ve, indicating a min. point.

At $x = 2$, $\frac{dy}{dx}$ changes from +ve to -ve, indicating a max. point.

At $x = 3$, $\frac{dy}{dx}$ changes from –ve to +ve, indicating a min. point.

The min. points are therefore (1, 0) and (3, 0), whilst the max. is at (2,1).



Method 2

$(x^2 - 4x + 3)^2 \geq 0$ and $(x^2 - 4x + 3)^2 = (x-1)^2(x-3)^2 = 0$ has roots at $x = 1$ & 3, so that there are minima at these two points.

For $x = 1 - t$, $y = x^2 - 4x + 3$ and hence $\text{y} = (x^2 - 4x + 3)^2$ increases as t increases, and similarly for $x = 3 + t$.

For $1 < x < 3$, $\text{y} = (x^2 - 4x + 3)^2$ attains a max. when

$x^2 - 4x + 3$ (which is negative in this range) is at a min. ; ie when

$x = 2$.