

**STEP/Differential Equations Q4 (15/6/23)**

Solve  $\frac{dy}{dx} = \frac{x^3+4y^3}{3xy^2}$ ,  $x > 0$

**Solution**

Let  $z = \frac{y}{x}$ , so that  $\frac{dy}{dx} = z + x \frac{dz}{dx}$ , as in (i).

$$\text{Then } z + x \frac{dz}{dx} = \frac{1}{3z^2} + \frac{4z}{3}$$

$$\text{and } x \frac{dz}{dx} = \frac{1}{3z^2} + \frac{z}{3}$$

$$\text{so that } 3 \int \frac{1}{\frac{1}{z^2} + z} dz = \int \frac{1}{x} dx$$

$$\text{and } \ln x = \int \frac{3z^2}{1+z^3} dz = \ln(1 + z^3) + \ln C$$

$$\Rightarrow x = C(1 + z^3) \quad [C > 0]$$

$$\Rightarrow \left(\frac{y}{x}\right)^3 = Ax - 1 \quad [A = \frac{1}{C}]$$

$$\Rightarrow y^3 = (Ax - 1)x^3$$