STEP/Differential Equations Q3 (15/6/23)

Show that $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$ can potentially be solved by making a substitution.

Solution

Let $z = \frac{y}{x}$, so that $y = xz$ and $\frac{dy}{dx} = z + x\frac{dz}{dx}$

So $\frac{dy}{dx} = f(\frac{y}{x})$ becomes $z + x\frac{dz}{dx} = f(z)$

and $\int \frac{1}{f(z)-z}dz = \int \frac{1}{x}\,dx$

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