

**STEP/Differential Equations Q1 (15/6/23)**

Solve  $\frac{dy}{dx} = x + y$  by:

- (a) finding an integrating factor
- (b) making the substitution  $z = x + y$

**Solution**

$$(a) \frac{dy}{dx} = x + y \Rightarrow \frac{dy}{dx} - y = x$$

$$\text{I.F.} = \exp \left\{ \int -1 \, dx \right\} = e^{-x}$$

$$\text{Then } e^{-x} \frac{dy}{dx} - e^{-x} y = x e^{-x}$$

$$\Rightarrow \frac{d}{dx} (y e^{-x}) = x e^{-x}$$

$$\Rightarrow y e^{-x} = \int x e^{-x} \, dx = x(-e^{-x}) - \int -e^{-x} \, dx = -x e^{-x} - e^{-x} + C$$

$$\Rightarrow y = C e^x - 1 - x$$

$$(b) \frac{dy}{dx} = x + y \Rightarrow \frac{d}{dx} (z - x) = z$$

$$\Rightarrow \frac{dz}{dx} - 1 = z$$

$$\Rightarrow \frac{dz}{dx} = z + 1$$

$$\Rightarrow \int \frac{1}{z+1} \, dz = \int dx$$

$$\Rightarrow \ln|z + 1| = x - \ln C$$

$$\Rightarrow C(z + 1) = e^x$$

$$\Rightarrow y = z - x = A e^x - 1 - x$$