STEP/Curve Sketching Q6 (17/6/23)

Sketch $y = \frac{e^x}{x}$

Solution

Vertical asymptote at $x = 0$

When $x = \delta$ (where δ is a small positive number), $y > 0$;

and when $x = -\delta$, $y < 0$.

Existence of horizontal asymptote

As $x \to \infty$, $y \to \infty$, and as $x \to -\infty$, $y \to 0^-$

Stationary points

$\frac{dy}{dx} = \frac{xe^x - e^x}{x^2} = e^x \frac{(x-1)}{x^2}$

So there is a stationary point at $x = 1$, when $y = e$.

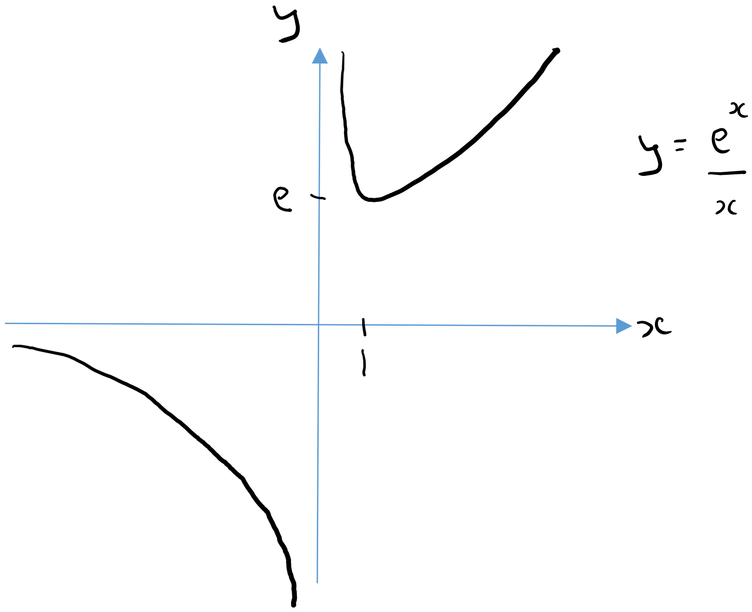
$\frac{d^2y}{dx^2} = e^x \frac{(x-1)}{x^2} + e^x(-x^{-2} + 2x^{-3}) = e^x \frac{(x^2-2x+2)}{x^3}$

When $x = 1, \frac{d^2y}{dx^2} > 0$, so that there is a minimum at $(1, e)$.

Gradient

Also, $x^2 - 2x + 2 = (x-1)^2 + 1 > 0$, so that $\frac{d^2y}{dx^2} > 0$ (ie

increasing gradient) for $x > 0$, and $\frac{d^2y}{dx^2} < 0$ for $x < 0$


y
e
1
x
y = e^x / x