

STEP/Curve Sketching Q5 (14/6/23)

Sketch $y = \frac{x}{\sqrt{x^2+p}}$, where p is a positive constant, for $x \geq 0$

Solution

Writing $f(x) = \frac{x}{\sqrt{x^2+p}}$,

$f(0) = 0$ and $f(x) \rightarrow 1^-$ as $x \rightarrow \infty$

$$f(x) = \frac{x}{\sqrt{x^2+p}} \Rightarrow f'(x) = \frac{\sqrt{x^2+p} - x \cdot \frac{1}{2}(x^2+p)^{-\frac{1}{2}} \cdot 2x}{x^2+p}$$

$$= \frac{(x^2+p) - x^2}{(x^2+p)^{\frac{3}{2}}} = \frac{p}{(x^2+p)^{\frac{3}{2}}} > 0 \text{ for } x \geq 0$$

And $f''(x) = p \left(-\frac{3}{2}\right) (x^2+p)^{-\frac{5}{2}} (2x) < 0$ for $x > 0$

