STEP/Curve Sketching Q3 (14/6/23)

Sketch the following:

(i) $y = ln\,(1 - x)$

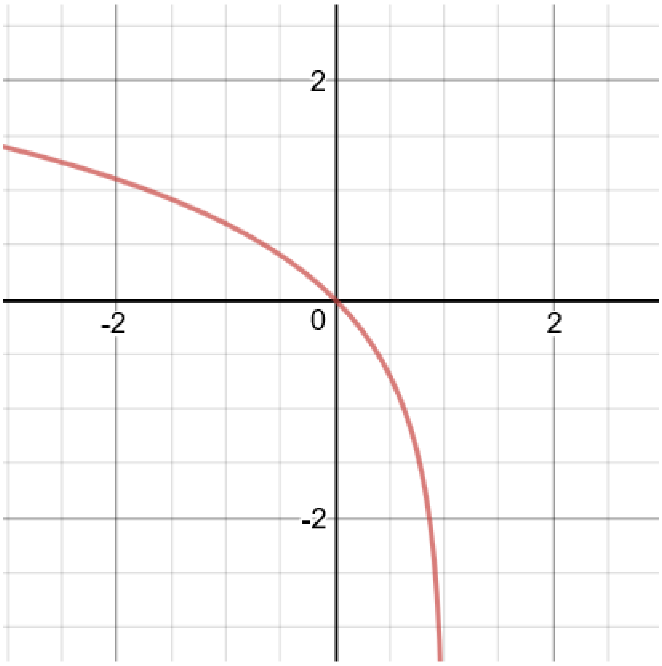
(ii) $y = ln(x^2 - 1)$

(iii) $y = ln|x^2 - 1|$

Solution

(i) $y = \ln(1-x)$ is the reflection in $x = \frac{1}{2}$ of $y = lnx$

[$y = lnx \rightarrow y = \ln(-x)$ is a reflection in the y-axis (note that the domain changes to negative x); then $\ln(-x) \rightarrow \ln(-[x-1]) =$ $\ln(1-x)$ is a translation of 1 to the right, which can be seen to be a reflection in $x = \frac{1}{2}$; also, compare with $y = \sin(\pi - x)$, which is the reflection in $x = \frac{\pi}{2}$ of $y = sinx$]



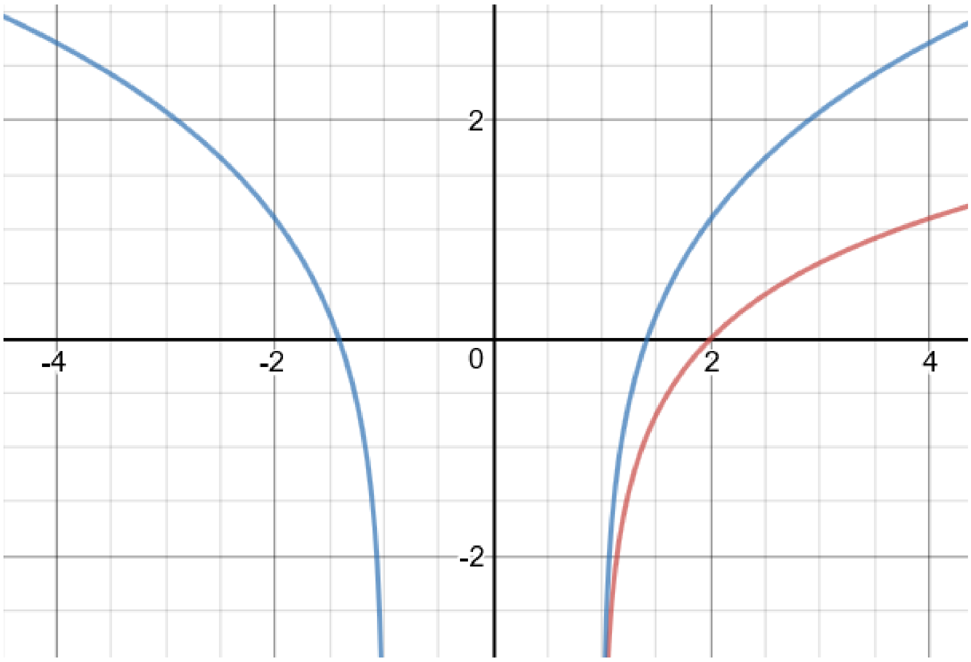
$y = \ln(1-x)$

(ii) $y = \ln(x^2 - 1)$ is an even function; ie it's symmetric about the y-axis. It is undefined for $-1 \leq x \leq 1$

The right-hand branch can be obtained from $y = \ln(x-1)$: For $x > 1$, $y = f(x^2)$ will be a compressed version of $y = f(x)$, with equality as $x \rightarrow 1$ [eg to obtain the point $(2, f(2^2))$, we start at

$(2,0)$ on the x-axis, then look to the right to obtain $(2^2,0)$, then up to the curve $y=f(x)$, to find the point $(2^2, f(2^2))$, which we drag to the left, to give $(2, f(2^2))$; thus the process is similar to a stretch of scale factor k, to obtain $y=f(kx)$ from $y=f(x)$, where $k>1$ (though with equality when $x=1$, rather than

$x=0$).]



$$y=\ln(x-1) \;\; \& \;\; y=\ln(x^2-1)$$

(iii) $y=ln|x^2-1|$

For $|x|>1$, $ln|x^2-1| = \ln(x^2-1)$

For $x=1$, $ln|x^2-1|$ is undefined

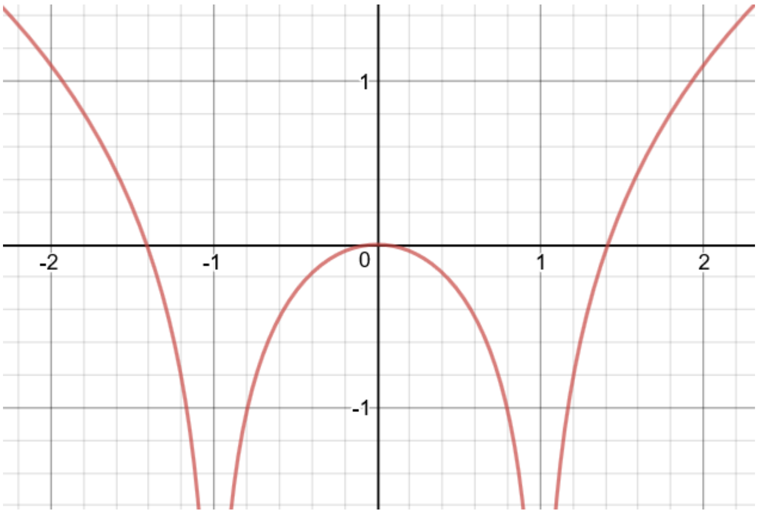
For $|x|<1$, $ln|x^2-1| = \ln(1-x^2)$

$y=ln(1-x^2)$ is an even function; ie it's symmetric about the y-axis. We need therefore only consider the curve for $0 \le x < 1$.

For $0 \le x < 1$, $y = ln(1 - x^2)$ will be similar to $y = ln(1 - x)$.

For $x = \frac{1}{2}$, for example, the y-coordinate will be $ln\ (1 - \frac{1}{4})$; ie we are looking to the left (to obtain $x = \frac{1}{4}$), and dragging the graph of $y = ln(1 - x)$ back to the right. Thus $y = ln(1 - x^2)$ hugs the line $x = 1$ (and also $y = 0$) more than $y = ln(1 - x)$.

[Compare with the graphs $y = x^2$ and $y = x^4$, where the latter is 'squarer'.]



$y = ln|x^2 - 1|$