STEP/Counting Q9 (11/6/23)

Given 6 pairs of twins, in how many ways can they be placed in 3 teams of 4, such that no team contains any pair of twins?

Solution

Let the 6 pairs of twins be labelled $AaBb \ldots Ff$

Define team 1 to be the team containing A.

Team 1 might, for example, be $AbCd$.

The number of ways of choosing the 3 people to go with A is

$\binom{5}{3}$ [the number of ways of choosing 3 of the 5 remaining pairs]

$\times 2^3$ [as either twin could be chosen for each pair]

For the case where team 1 is $AbCd$, define team 2 to be the team containing E. [Had E been chosen for team 1, it would have been another letter that was chosen to define team 2]

One complete selection of teams would then be:

team 1: $AbCd$

team 2: $a\ \ c\ \ \ Ef$

team 3: B DeF

There is the following scope for choice:

$\binom{4}{2}$ ways of choosing the 2 people out of $aBcD$ to go in team 2; combined with the 2 ways of choosing either $f\ or\ F$ for team 2

Thus the overall total number of ways is:

$\binom{5}{3} \times 2^3 \times \binom{4}{2} \times 2 = 10 \times 8 \times 6 \times 2 = 960$

Note: For team 1, an alternative calculation is as follows:

The number of ways of filling the remaining 3 places in team 1 is $10 \times 8 \times 6$, if order is important; but, as order isn't important, we divide by 3!, to give 10×8, as before.