

STEP/Counting Q6 (11/6/23)

Five poorly-behaved pupils are required to sit in the front five places in a classroom. Angus insists on sitting next to Bruce, Chantal refuses to sit next to Deborah, and Emily is happy to sit anywhere. In how many different ways can they take their seats?

Solution

Let X represent A & B.

Suppose that X is to the left of E.

Then C & D can go in the spaces shown here: _X_E_

There are 3 choices for C, and then 2 choices for D, giving 6 possibilities.

Multiply by 2, to include cases where X is to the right of E; giving 12 possibilities.

Multiply by 2 again, as X could be AB or BA, giving 24 possibilities.

Alternative approach

Number of ways with no constraints on C & D, where A is ahead of B (eg XABXX):

4 (ways of placing A)

× 3! (ways of placing C, D & E)

= 24

Including cases where B is ahead of A gives $24 \times 2 = 48$ (1)

Permutations to be excluded, with A ahead of B and C ahead of D:

ABCDE, ABECD, EABCD, CDABE, CDEAB, ECDAB

giving a total of $6 \times 2 \times 2 = 24$ to be excluded (including cases where B is ahead of A and/or D is ahead of C) (2)

Hence, number of allowable ways = (1) – (2) = 24

[It is also possible to consider only situations of the form ABXXX (or BAXXX) and XABXX (or XBAXX), and multiply by 2 to cover the symmetrical situations where we start from the other end.]