STEP/Counting Q3 (11/6/23)

(i) 3 different sweets are to be shared amongst 5 children. In how many ways can this be done, if no child is to receive all 3 sweets?

(ii) What is the answer if the 3 sweets are indistinguishable?

Solution

(i) **Method 1**

There are 5^3 ways of allocating the children to the 3 sweets. Then deduct the 5 ways in which one child has all 3 sweets, to give

$5^3 - 5 = 120$

Method 2

The permissible cases are: ABC, AAB, ABA, BAA (where AAB means that sweets 1 & 2 go to child A & sweet 3 goes to child B).

The total number of possibilities is then:

$5(4)(3) + 5(4) + 5(4) + 5(4)$ [for AAB eg, there are 5 ways of choosing A, and then 4 ways of choosing B]

$= 60 + 20 + 20 + 20 = 120$

(ii) If the 3 sweets are indistinguishable, there are 2 permissible cases:

AAB and ABC (where AAB, for example, means that child A receives 2 sweets & child B receives 1 sweet)

The total number of such cases is 5×4 (5 ways of choosing A, and then 4 ways of choosing B) $+ \binom{5}{3} = 20 + 10 = 30$.