STEP/Counting Q2 (11/6/23)

Show that the number of ways of selecting r items from n (distinct) items, if repetitions are allowed and order is not important, is $\binom{n-1+r}{r}$

Solution

eg $n = 3, r = 3$

Let the items be A, B & C

Possibilities are:

AAA; BBB; CCC

AAB/AAC; BBA/BBC; CCA/CCB

ABC

Total of 10

Write BBC as |XX|X

(| indicates that we are moving on to the next letter, and XX indicates that we are choosing 2 items from the current letter)

Number of ways of choosing r positions for the X, out of $n - 1 + r$ possible positions $= \binom{n-1+r}{r}$

$[n = 3, r = 3: \binom{n-1+r}{r} = \binom{5}{3} = \binom{5}{2} = 10]$