STEP/Counting Q1 (11/6/23)

Prove that $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$

[where $\binom{n}{r}$ is written instead of ${}^{n}C_{r}$]

Solution

If r items are to be chosen from n items, then either the 1st item is included or it isn't.

If it is included, then there are $\binom{n-1}{r-1}$ ways of choosing the remaining $r-1$ items that are required.

If it isn't included, then there are $\binom{n-1}{r}$ ways of choosing the remaining r items that are required.

This gives a total of $\binom{n-1}{r-1}+\binom{n-1}{r}$ ways of choosing the r items.