STEP/Counting Q11 (15/3/24)

The random variable $X \sim B(3, \frac{1}{2})$.

(i) Find $P(X = 2 | X \geq 1)$

(ii) Find $P(X = 2 | 1st$ item in the Binomial trial is a success)

Solution

(i) **Method 1**

$$P(X = 2 | X \geq 1) = \frac{P(X=2 \ \& \ X \geq 1)}{P(X \geq 1)} = \frac{P(X=2)}{P(X \geq 1)}$$

$$= \frac{3\left(\frac{1}{2}\right)^3}{1 - P(X=0)} = \frac{\left(\frac{3}{8}\right)}{1 - \left(\frac{1}{2}\right)^3} = \frac{\left(\frac{3}{8}\right)}{\left(\frac{7}{8}\right)} = \frac{3}{7}$$

Method 2

$$P(X = 2 | X \geq 1) = \frac{No. \ of \ ways \ of \ obtaining \ 2 \ successes}{No. \ of \ ways \ of \ obtaining \ 1 \ or \ more \ successes}$$

(provided each way is equally likely)

$$= \frac{\#(SSF, SFS, FSS)}{Total \ no. \ of \ ways - no. \ of \ ways \ of \ obtaining \ 0 \ successes}$$

(where # denotes "the number of items in the given list"; not

standard notation)

$$= \frac{3}{2 \times 2 \times 2 - \#(FFF)} = \frac{3}{8-1} = \frac{3}{7}$$

[or $\dfrac{\binom{3}{2}}{\binom{3}{1} + \binom{3}{2} + \binom{3}{3}} = \dfrac{3}{3+3+1} = \dfrac{3}{7}$; $\binom{3}{2}$ being interpreted as the number

of ways of choosing 2 places for the Ss]

(ii)

Method 1

$P(X = 2 | 1st$ item in the Binomial trial is a success)

$= P(Y = 1)$, where $Y \sim B(2, \frac{1}{2})$

$= 2(\frac{1}{2})^2 = \frac{1}{2}$

[Note that the chance of obtaining 2 successes is greater when we are told that the 1st item was a success than when we are just told that (overall) at least one of the items was a success - as in (i).]

Method 2

$P(X = 2 | 1st$ item in the Binomial trial is a success)

$$= \frac{No.\ of\ ways\ of\ obtaining\ 2\ successes\ with\ the\ 1st\ item\ being\ a\ success}{No.\ of\ ways\ for\ the\ 1st\ item\ to\ be\ a\ success}$$

$$= \frac{2}{1 \times 2 \times 2} = \frac{1}{2}$$

[There are 2 possible places for the 2nd success, and $1 \times 2 \times 2$ is the number of ways of choosing items for the 3 places, with the 1st item being a success]