STEP/Counting: Exercises - Overview (15/3/24)

Q1

Prove that $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$

[where $\binom{n}{r}$ is written instead of ^nC_r]

Q2

Show that the number of ways of selecting r items from n (distinct) items, if repetitions are allowed and order is not important, is $\binom{n-1+r}{r}$

Q3

(i) 3 different sweets are to be shared amongst 5 children. In how many ways can this be done, if no child is to receive all 3 sweets?

(ii) What is the answer if the 3 sweets are indistinguishable?

Q4

The following books are on a bookshelf: 4 novels, 3 history books, 2 biographies and 1 dictionary. In how many ways can they be arranged if the novels have to be together, and similarly for the history books and biographies?

Q5

2 boys and 3 girls are to sit in a row. How many arrangements are there in which the 2 boys are not next to each and the 3 girls are also not next to each other?

Q6

Five poorly-behaved pupils are required to sit in the front five places in a classroom. Angus insists on sitting next to Bruce, Chantal refuses to sit next to Deborah, and Emily is happy to sit anywhere. In how many different ways can they take their seats?

Q7

6 people (labelled A-F) are to be seated round a circular table. How many seating arrangements are possible if B and E are not to sit next to each other?

Q8

A 4-digit password is made up of numbers from 0 to 4, where the numbers can be repeated, but have to be ordered from largest to smallest. Show that there are 70 possible passwords.

Q9

Given 6 pairs of twins, in how many ways can they be placed in 3 teams of 4, such that no team contains any pair of twins?

Q10

(i) How many ways are there of writing the pair of integers (i, j), given that $1 \le i < j \le n$?

(ii) Show that there are $n(n-1)(n-2)$ ways of writing the integers (i, j, k, l), given that $1 \le i < j \le n$ and

$1 \le k < l \le n$, and such that exactly one of the numbers is

repeated (eg (1,2,1,3) or (1,2,2,3), but not (1,2,1,2))

Q11

The random variable $X \sim B(3, \frac{1}{2})$.

(i) Find $P(X = 2 | X \ge 1)$

(ii) Find $P(X = 2 | 1st$ item in the Binomial trial is a success)