# STEP/Algebra Q4 (13/6/23)

If $\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$ , $\phi = \frac{1}{\sqrt{1-\left(\frac{u}{c}\right)^2}}$ and $w = \frac{u+v}{1+\frac{uv}{c^2}}$,

show that $\left(1 + \frac{uv}{c^2}\right)\gamma\phi = \frac{1}{\sqrt{1-\left(\frac{w}{c}\right)^2}}$

**Solution**

The required result is equivalent to

$$\left(1+\frac{uv}{c^2}\right)^2\left(1-\left(\frac{w}{c}\right)^2\right)=\left(1-\left(\frac{u}{c}\right)^2\right)\left(1-\left(\frac{v}{c}\right)^2\right)$$

or $\left(1+\frac{uv}{c^2}\right)^2\left(1-\left(\frac{w}{c}\right)^2\right)-\left(1-\left(\frac{u}{c}\right)^2\right)\left(1-\left(\frac{v}{c}\right)^2\right)=0$

$$LHS=\left\{1+\frac{2uv}{c^2}+\frac{(uv)^2}{c^4}\right\}-\frac{(u+v)^2}{c^2}-\{1-\frac{u^2}{c^2}-\frac{v^2}{c^2}+\frac{(uv)^2}{c^4}\}$$

$=0$, as required.

[This is a result from Special Relativity: if spaceship C is seen by spaceship B to be moving away from it at speed $v$, and spaceship B is seen by spaceship A to be moving away from it (in the same direction as previously) at speed $u$, then Newtonian Physics gives the speed of C relative to A as just $u+v$, but according to Special Relativity it is $w$.

$\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$ is the Lorentz factor associated with changes in measurements of time and length for an object moving at relative speed $v$.]