

STEP/Algebra Q3 (13/6/23)

Given that $\frac{bc-a}{1-c} = 7$ & $\frac{b^2c-a^2}{1-c} = 51$,

show that $\frac{a+7}{a^2+51} = \frac{b+7}{b^2+51}$

Solution

$$\frac{bc-a}{1-c} = 7 \Rightarrow bc - a = 7 - 7c \Rightarrow c(b+7) = 7+a$$

$$\Rightarrow c = \frac{a+7}{b+7}$$

and replacing a, b & 7 with a^2, b^2 & 51 gives $c = \frac{a^2+51}{b^2+51}$

so that $\frac{a+7}{b+7} = \frac{a^2+51}{b^2+51}$ and hence $\frac{a+7}{a^2+51} = \frac{b+7}{b^2+51}$ (since $a^2 + 51$

& $b^2 + 51$ are both non-zero)