

STEP/Algebra Q2 (13/6/23)

Solve the equation $\sqrt{2x + 3} + \sqrt{x + 1} = \sqrt{7x + 4}$

Solution

$$\sqrt{2x+3} + \sqrt{x+1} = \sqrt{7x+4} \quad (*)$$

$$\Rightarrow (2x+3) + 2\sqrt{(2x+3)(x+1)} + (x+1) = 7x+4$$

(incl. possible spurious sol'ns)

$$\Rightarrow 2\sqrt{(2x+3)(x+1)} = 4x$$

$$\Rightarrow (2x+3)(x+1) = 4x^2$$

$$\Rightarrow 2x^2 - 5x - 3 = 0$$

$$\Rightarrow (2x+1)(x-3) = 0$$

$$\Rightarrow x = -\frac{1}{2} \text{ or } 3$$

But only $x = 3$ satisfies (*)

$$[x = -\frac{1}{2} \text{ is a sol'n of } 2\sqrt{(2x+3)(x+1)} = -4x]$$