STEP/Algebra Q1 (13/6/23)

Solve the equation $x - \sqrt{x} = 6$

Solution

Let $f(x) = x - \sqrt{x} - 6$

$f(x) = 0 \Rightarrow x - 6 = \sqrt{x}$

$\Rightarrow (x-6)^2 = x$, but this may include spurious solutions

[of $x - 6 = -\sqrt{x}$]

$\Rightarrow x^2 - 13x + 36 = 0$

$\Rightarrow (x-9)(x-4) = 0$

$\Rightarrow x = 9 \; or \; x = 4$

$f(9) = 0 \;\; \& \, f(4) = -4$

Thus the only solution is $x = 9$

[Let $g(x) = x + \sqrt{x} - 6 = 0$

Then $g(x) = 0 \Rightarrow (x-6)^2 = x$ as well

$g(9) \neq 0, and \; g(4) = 0$]

Alternatively: Let $y = \sqrt{x}$, so that

$x - \sqrt{x} - 6 = 0 \Rightarrow y^2 - y - 6 = 0$

$\Rightarrow (y+2)(y-3) = 0$

$\Rightarrow y = -2$ (reject as $\sqrt{\text{x}}$ must be ≥ 0) $or \; y = 3$