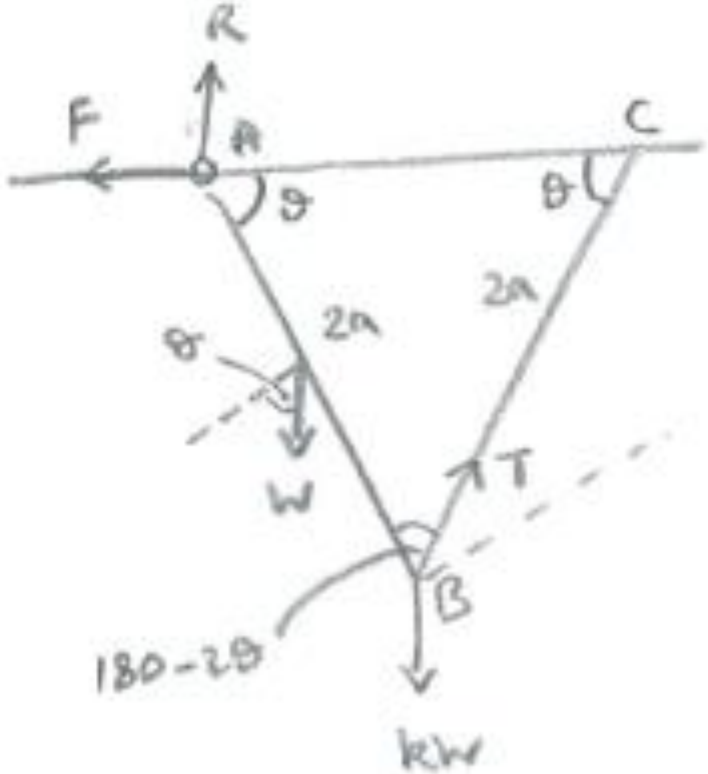
STEP 2019, P2, Q10 - Solution (2 pages; 5/2/21)

(i)



Taking moments about A in respect of the rod AB,

$-Wcos\theta.a - kWcos\theta.2a + Tcos(90 - [180 - 2\theta]).2a = 0$,

as the rod is in rotational equilibrium

$\Rightarrow 2Tcos(2\theta - 90) = Wcos\theta(1 + 2k)$

$\Rightarrow \frac{T}{W} = \frac{cos\theta(1+2k)}{2\cos(90-2\theta)} = \frac{cos\theta(1+2k)}{2sin(2\theta)} = \frac{1+2k}{4sin\theta}$

The string will break if $T > W$,

ie if $\frac{1+2k}{4sin\theta} > 1$ or $2k + 1 > 4sin\theta$, as required

(ii) Applying N2L to the rod AB, resolving vertically:

$R + Tcos(90 - \theta) = W + kW$

$\Rightarrow R + Tsin\theta = (k+1)W \quad (1)$

Resolving horizontally:

$F = Tsin(90 - \theta) = Tcos\theta \quad (2)$

And, from (i), $\frac{T}{W} = \frac{1+2k}{4sin\theta}$, so that $W = \frac{4Tsin\theta}{1+2k}$

Subst. into (1) then gives $R + Tsin\theta = (k+1)\frac{4Tsin\theta}{1+2k}$

$\Rightarrow R = Tsin\theta\left(\frac{4k+4}{1+2k} - 1\right) = \frac{Tsin\theta(4k+4-1-2k)}{1+2k} = \frac{Tsin\theta(2k+3)}{1+2k} \quad (3)$

Then, from (2) & (3), $\frac{F}{R} = \frac{cos\theta(1+2k)}{sin\theta(2k+3)}$

The ring will slip if $F > \mu R$;

ie if $\frac{cos\theta(1+2k)}{sin\theta(2k+3)} > \mu$ or $2k + 1 > (2k+3)\mu tan\theta$, as required

(iii) From (i), the string will break if $k > \frac{4sin\theta - 1}{2}$

From (ii), the ring will slip if $2k(1 - \mu tan\theta) > 3\mu tan\theta - 1$

ie if $k > \frac{3\mu tan\theta - 1}{2(1 - \mu tan\theta)}$

So the ring will slip before the string breaks if $\frac{3\mu tan\theta - 1}{2(1-\mu tan\theta)} < \frac{4sin\theta - 1}{2}$

ie if $3\mu tan\theta - 1 < (1 - \mu tan\theta)(4sin\theta - 1)$

$\Leftrightarrow \mu(3tan\theta + 4tan\theta sin\theta - tan\theta) < 4sin\theta$

$\Leftrightarrow \mu < \frac{4sin\theta}{2tan\theta + 4tan\theta sin\theta} = \frac{2cos\theta}{1+2sin\theta}$, as required