STEP 2019, P1, Q9 - Solution (3 pages; 2/7/20)

[To make any progress with this question, it has to be assumed that the ladder extends beyond the edge of the box (as shown in the diagram below). This isn't entirely obvious from the wording of the question. Generally, the angrier the examiner's comments are, the more ambiguous is the question!]

(i) Fig. 1 shows the set-up, Fig. 2 is the force diagram for the box, and Fig. 3 is the force diagram for the ladder.

Note that, as $h = btan\alpha$, $tan\alpha = \frac{h}{b}$, so that the base of the ladder is a distance b from the box.

Also, the force exerted by the ladder on the box is also R, by N3L.

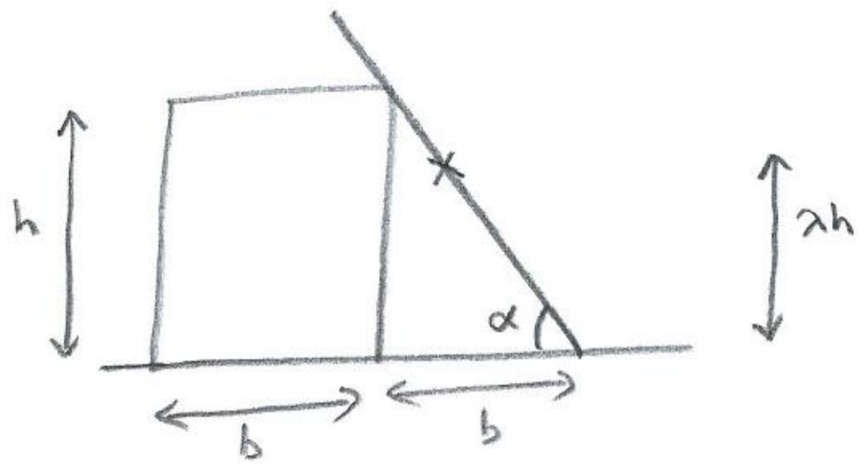
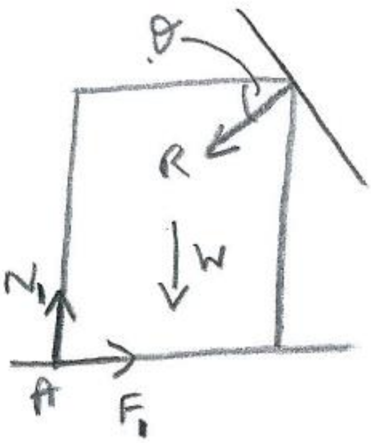


Fig. 1



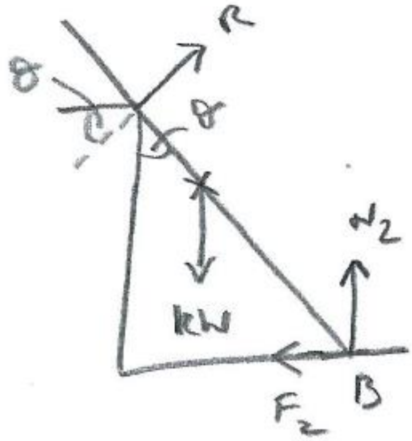


Fig. 2

Fig. 3

The ladder is in rotational equilibrium, and so taking moments about B gives:

$$-Rcos\theta.h - Rsin\theta.b + kW.\frac{\lambda h}{tan\alpha} = 0 \quad (1)$$

And $tan\theta = \frac{b}{h}$, so that (1) $\Rightarrow$

$$R = \frac{k\lambda Wb}{cos\theta.h + sin\theta.b} \quad (\text{as } h = btan\alpha)$$

$$= \frac{k\lambda Wbcos\alpha}{cos\theta.bsin\alpha + sin\theta.bcos\alpha}$$

$$= \frac{k\lambda Wcos\alpha}{\sin(\theta+\alpha)}$$

$= k\lambda Wcos\alpha$ (as $\theta + \alpha = 90°$), as required.

(ii) As the box is about to topple (about A), the normal reaction at the base acts at A. There is still rotational equilibrium, and so taking moments about A gives:

$$Rcos\theta.h - Rsin\theta.b - W\left(\frac{b}{2}\right) = 0$$

$\Rightarrow k\lambda Wcos\alpha(cos\theta.btan\alpha - sin\theta.b) - W\left(\frac{b}{2}\right) = 0$, from (i)

$$\Rightarrow k\lambda cos\alpha(cos\theta.tan\alpha - sin\theta) - \frac{1}{2} = 0$$

$$\Rightarrow 2k\lambda(cos\alpha sin\theta - cos\theta sin\alpha) + 1 = 0 \quad (2)$$

Now $cos\theta = sin\alpha$ and $sin\theta = cos\alpha$,

so that (2) $\Rightarrow 2k\lambda(cos^2\alpha - sin^2\alpha) + 1 = 0$

$\Rightarrow 2k\lambda cos2\alpha + 1 = 0$, as required.

(iii) Given that the box doesn't slip, $F_1 \leq \mu N_1$

Resolving horiz. and vert.,

$F_1 = Rcos\theta$ & $N_1 = W + Rsin\theta$

Then $\mu \geq \frac{Rcos\theta}{W+Rsin\theta} = \frac{k\lambda Wcos\alpha cos\theta}{W+k\lambda Wcos\alpha sin\theta}$, from (i)

$= \frac{cos\alpha sin\alpha}{\frac{1}{k\lambda}+cos\alpha cos\alpha}$

$= \frac{\frac{1}{2}sin2\alpha}{-2cos2\alpha+\frac{1}{2}(1+cos2\alpha)}$, from (ii)

$= \frac{sin2\alpha}{-4cos2\alpha+1+cos2\alpha} = \frac{sin2\alpha}{1-3cos2\alpha}$, as required.