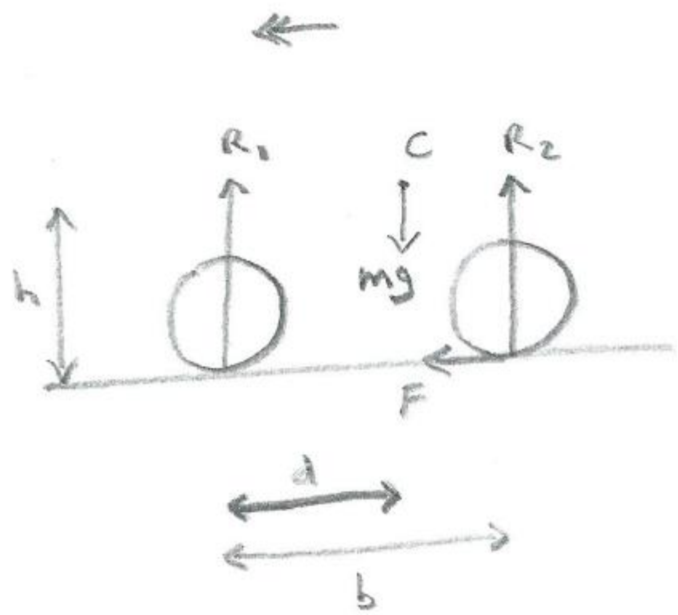
STEP 2018, P2, Q11 - Solution (3 pages; 14/11/18)

The engine applies internal forces on the wheel (a couple; ie the equivalent of a pair of equal and opposite forces, with different lines of action, causing rotation). By Newton's 3rd law, the wheel applies equal and opposite forces on the engine. The overall effect is two sets of forces that cancel out (note that, although these forces act on different objects - ie the engine and the wheel - both of these objects are part of the motorbike, and so the net resultant force on the motorbike is zero). It is in fact the friction at the ground that causes the motorbike to accelerate (see "Rolling wheels" for further details), although the friction only arises because of the rotation of the wheel (caused by the engine).

The force diagram for the motorbike only shows the external forces.



[C is the centre of mass]

$R_1 + R_2 = mg$ (1)

$F = ma$ (2) , where a is the acceleration of the motorbike

$M(C){:} - R_1 d + R_2(b-d) - Fh = 0$ (3)

Also, $F \leq \mu R_2$ (4)

[as we are told that the sum of the moments about C is zero]

Suppose that the rear wheel slips when the acceleration is a_R , and that the front wheel loses contact when the acceleration is a_F.

[This approach also helps with the subsequent parts of the question.]

The rear wheel will slip before the front wheel loses contact if

$a_R < a_F$

Then $ma_R = \mu R_2$

and, from (3), $-(0)d + R_2(b-d) - ma_F h = 0$

(as $F = ma_F$ when $R_1 = 0$)

and so $a_R = \frac{\mu R_2}{m}$, whilst $a_F = \frac{R_2(b-d)}{mh}$

Then $a_R < a_F \Leftrightarrow \frac{a_R}{a_F} < 1 \Leftrightarrow \frac{\mu h}{b-d} < 1 \Leftrightarrow \mu < \frac{b-d}{h}$ QED

[2nd part]

If $\mu < \frac{b-d}{h}$ and the rear wheel doesn't slip, the maximum acceleration is $a_R = \frac{\mu R_2}{m}$

From (1), $R_1 + R_2 = mg$, so that $a_R = \frac{\mu g R_2}{R_1 + R_2}$

rtp [result to prove]: $\frac{R_1 + R_2}{R_2} = \frac{b - \mu h}{d}$

From (3), $-R_1 d + R_2(b-d) - ma_R h = 0$,

and, as $a_R = \frac{\mu R_2}{m}$, this gives

$-R_1 d + R_2(b-d) - \mu R_2 h = 0$,

so that $-\frac{R_1}{R_2}d + (b - d) - \mu h = 0$

and $\frac{R_1+R_2}{R_2} = \frac{R_1}{R_2} + 1 = \frac{(b-d-\mu h)}{d} + 1 = \frac{b-\mu h}{d}$ QED

[3rd part]

We have shown that $a_R < a_F \Leftrightarrow \mu < \frac{b-d}{h}$,

so if $\mu \geq \frac{b-d}{h}$ it follows that $a_F \leq a_R$

and the maximum acceleration is a_F (when $R_1 = 0$)

From (3), $-(0)d + R_2(b - d) - ma_F h = 0$

so that $a_F = \frac{R_2(b-d)}{mh}$

From (1), $R_1 + R_2 = mg$, so that $R_2 = mg$ when $R_1 = 0$

and $a_F = \frac{g(b-d)}{h}$ is the required maximum acceleration.