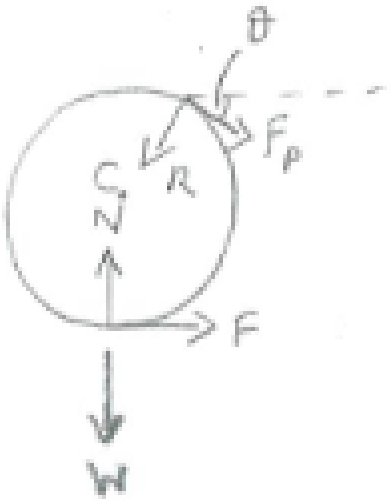
STEP 2017, P2, Q9 - Solution (4 pages; 18/2/21)

(i) 1st part

Create a force diagram for the left-hand cylinder (see below).



Taking moments about C, $rF_P = rF$ (as the cylinder is in rotational equilibrium), so that $F_P = F$.

Resolving forces horizontally, $Rsin\theta = F_P cos\theta + F = F(1 + cos\theta)$,

as required.

2nd part

As the plank hasn't slipped, $F_P \leq \frac{1}{2}R$, so that $\frac{F}{R} \leq \frac{1}{2}$

Then, $Rsin\theta = F(1 + cos\theta) \Rightarrow \frac{sin\theta}{1+cos\theta} \leq \frac{1}{2}$,

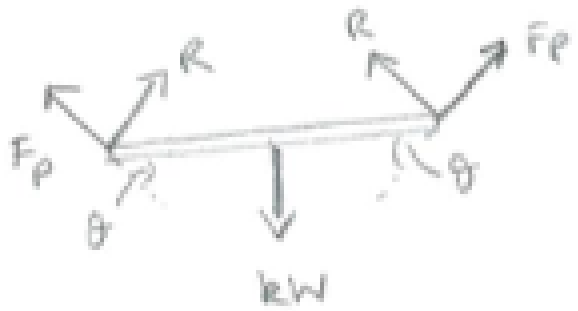
so that $2sin\theta \leq 1 + cos\theta$, as required.

(ii) 1st part

Resolving forces vertically,

$W + Rcos\theta + F_P sin\theta = N \quad (1)$

Create a force diagram for the plank (see below).



Resolving forces vertically,

$kW = 2(Rcos\theta + F_P sin\theta)$ (2)

Then, eliminating W from (1) & (2),

$N - Rcos\theta - Fsin\theta = \frac{2}{k}(Rcos\theta + F_P sin\theta)$

Then, as $Rsin\theta = F(1 + cos\theta)$,

$N = \frac{F(1+cos\theta)cos\theta(1+\frac{2}{k})}{sin\theta} + (1 + \frac{2}{k})(Fsin\theta)$

$= \frac{F(1+cos\theta)\left(1+\frac{2}{k}\right)}{sin\theta}\{cos\theta + \frac{sin^2\theta}{(1+cos\theta)}\}$

$= \frac{F(1+cos\theta)\left(1+\frac{2}{k}\right)}{sin\theta} \cdot \frac{cos\theta+cos^2\theta+sin^2\theta}{1+cos\theta}$

$= \frac{F(1+cos\theta)\left(1+\frac{2}{k}\right)}{sin\theta}$, as required. (3)

2nd part

Condition for cylinder not to slip is: $F \leq \frac{1}{2}N$ or $\frac{N}{F} \geq 2$

3rd part

From (3), $\frac{N}{F} = \frac{(1+cos\theta)\left(1+\frac{2}{k}\right)}{sin\theta} > \frac{1+cos\theta}{sin\theta} \geq \frac{2sin\theta}{sin\theta}$ (from (i)) $= 2$

Thus $\frac{N}{F} \geq 2$ for all θ.

(iii) **1st part**

$2sin\theta \leq 1 + cos\theta \Rightarrow 4sin^2\theta \leq 1 + cos^2\theta + 2cos\theta$ (as $sin\theta > 0$)

$\Rightarrow 4(1 - cos^2\theta) \leq 1 + cos^2\theta + 2cos\theta$

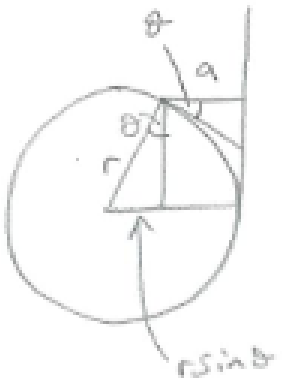
$\Rightarrow 5cos^2\theta + 2cos\theta - 3 \geq 0$

$\Rightarrow (5cos\theta - 3)(cos\theta + 1) \geq 0$

$\Rightarrow cos\theta \geq \frac{3}{5}$ (as $cos\theta > 0$, so that $cos\theta + 1 > 0$)

$\Rightarrow sin\theta \leq \sqrt{1 - \left(\frac{3}{5}\right)^2} = \frac{4}{5}$

2nd part



$rsin\theta + a = r \Rightarrow r(1 - sin\theta) = a$

$\Rightarrow \frac{a}{r} = 1 - sin\theta \geq 1 - \frac{4}{5} = \frac{1}{5}$

$\Rightarrow 5a \geq r$, as required.