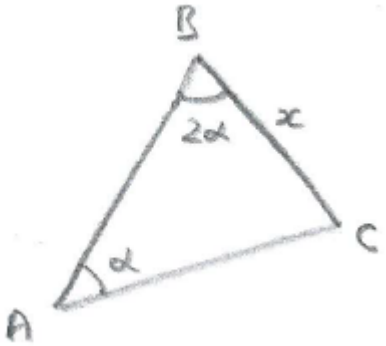
STEP 2015, Paper 2, Q2 Solution (2 pages; 20/5/18)

1st part



$$\frac{x}{sin\alpha} = \frac{AB}{sin(\pi-3\alpha)} \Rightarrow AB = \frac{xsin(\pi-\alpha-2\alpha)}{sin\alpha}$$

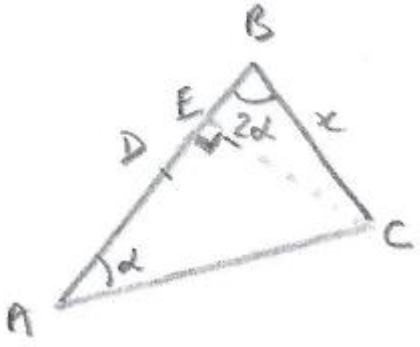
$$= \frac{x}{sin\alpha}(\sin(\pi-\alpha)\cos(2\alpha) - \cos(\pi-\alpha)\sin(2\alpha))$$

$$= \frac{x}{sin\alpha}(\sin\alpha\,(cos^2\alpha - sin^2\alpha) + cos\alpha(2sin\alpha cos\alpha)$$

$$= x(1 - 2sin^2\alpha + 2(1 - sin^2\alpha))$$

$$= x(3 - 4sin^2\alpha) \text{ QED}$$

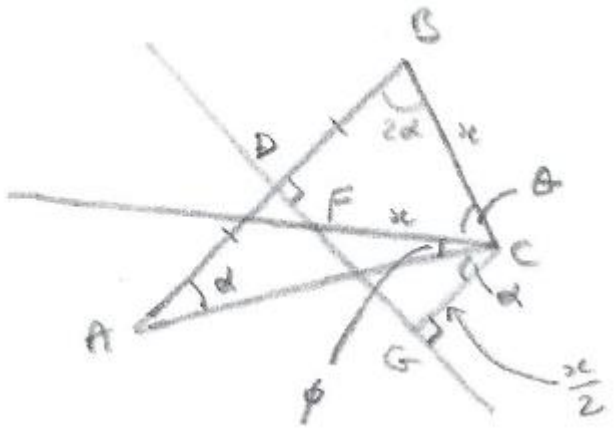
2nd part



$$BE = xcos(2\alpha)$$

$$DE = \frac{1}{2}AB - BE = \frac{x}{2}(3 - 4sin^2\alpha) - x(1 - 2sin^2\alpha) = \frac{x}{2}$$

3rd part



rtp (result to prove): $\theta = \frac{2}{3}(\pi - 3\alpha) = \frac{2\pi}{3} - 2\alpha$

As $DE = \frac{x}{2}$, $CG = \frac{x}{2}$ also

And CG is parallel to AB, so that $\angle GCA = \alpha$

Then $cos(\angle CGF) = \frac{(x/2)}{x} = \frac{1}{2} \Rightarrow \angle CGF = \frac{\pi}{3}$

As DBCG is a quadrilateral, $\angle GCB = \pi - 2\alpha$

So $\theta + \phi + \alpha = \pi - 2\alpha$ and $\alpha + \phi = \frac{\pi}{3}$

$\Rightarrow \theta + \frac{\pi}{3} = \pi - 2\alpha \Rightarrow \theta = \frac{2\pi}{3} - 2\alpha$ QED