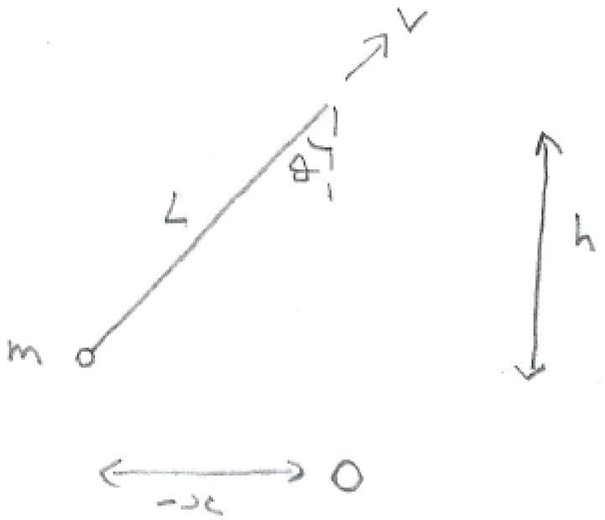
STEP 2015, Paper 2, Q10 Solution (3 pages; 21/5/18)

1st part



x is the x coordinate from O, with left to right being positive

$tan\theta = \frac{-x}{h} \Rightarrow \frac{dx}{dt} = -hsec^2\theta\frac{d\theta}{dt}$ (1)

Also, $\frac{dL}{dt} = -V$ (2)

and $Lcos\theta = h,$ (2a)

so that (differentiating wrt L)

$cos\theta + L(-sin\theta)\frac{d\theta}{dL} = 0$, and hence $\frac{d\theta}{dL} = \frac{cot\theta}{L}$ (3)

Then (1) $\Rightarrow \frac{dx}{dt} = -hsec^2\theta\frac{d\theta}{dL}\frac{dL}{dt} = -hsec^2\theta(\frac{cot\theta}{L})(-V)$,

from (2) & (3)

So $\frac{dx}{dt} = \frac{hVsec^2\theta cot\theta}{hsec\theta}$, as $L = hsec\theta$ (from (2a))

$= Vcosec\theta$

2nd part

$\frac{d^2x}{dt^2} = \frac{d}{dt}(Vcosec\theta) = V\frac{d}{dt}((sin\theta)^{-1})$

$= V(-(sin\theta)^{-2}cos\theta)\frac{d\theta}{dt}$

$= -V(\frac{cos\theta}{sin^2\theta})\frac{d\theta}{dL}\frac{dL}{dt}$

$= -V(\frac{cos\theta}{sin^2\theta})(\frac{cot\theta}{L})(-V)$, from (2) & (3) again

$= \frac{V^2cos^2\theta}{Lsin^3\theta}$

$= \frac{V^2cos^3\theta}{hsin^3\theta}$, as $Lcos\theta = h$, from (2a)

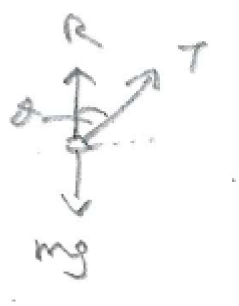
$= \frac{V^2cot^3\theta}{h}$

3rd part

By N2L, $Tsin\theta = m\frac{d^2x}{dt^2} = m\frac{V^2cot^3\theta}{h}$,

so that $T = \frac{mV^2cot^3\theta}{hsin\theta}$

4th part



Particle will leave the floor when $R = 0$

$R + Tcos\theta = mg$

Then $R = 0 \Rightarrow mg - Tcos\theta = 0$

$$\Rightarrow mg - \frac{mV^2 cot^3\theta cos\theta}{hsin\theta} = 0$$

$$\Rightarrow g = \frac{V^2 cot^4\theta}{h}$$

$$\Rightarrow tan^4\theta = \frac{V^2}{gh} \quad \text{QED}$$