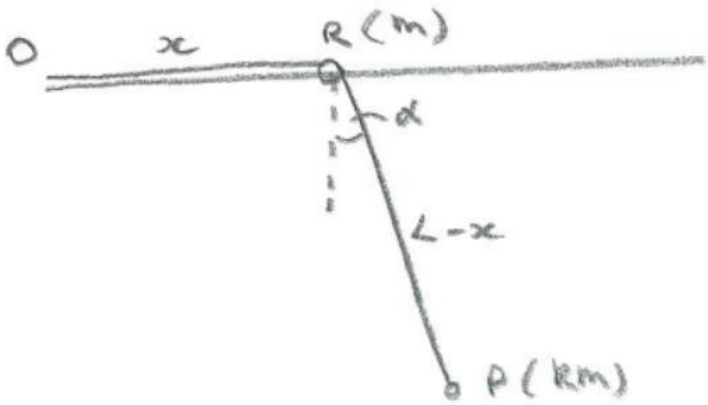
STEP 2014, P2, Q11 - Solution (2 pages; 6/4/21)



[This configuration might be achieved if the part of the string RP (but excluding the ring and the particle) is resting on a surface inclined at an angle α with the vertical. This surface would thus introduce the constraint (or boundary condition) $\alpha = constant$, without affecting the eq'n of motion.]

Coordinates of P are $(x + (L - x)sin\alpha, -(L - x)cos\alpha)$

(i) **1st part**

By N2L, $Tcos\alpha - kmg = km\ \frac{d^2}{dt^2}\{-(L - x)cos\alpha)\}$

$= km\ddot{x}cos\alpha$, as required.

2nd part

For P, $-Tsin\alpha = km\ \frac{d^2}{dt^2}\{x + (L - x)sin\alpha\}$

$= km\ddot{x}(1 - sin\alpha)$

For R, $Tsin\alpha - T = m\ddot{x}$

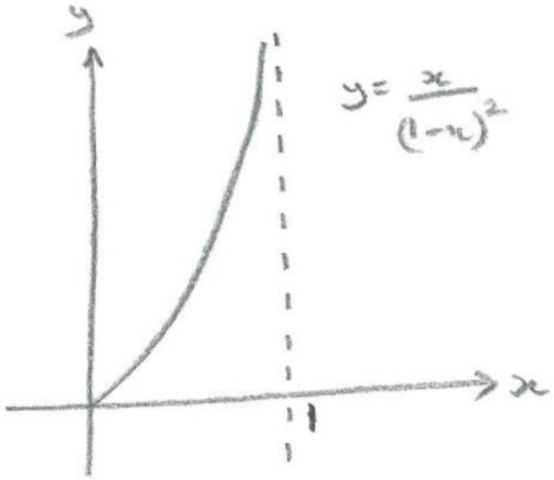
(ii) **1st part**

From the 2nd part of (i),

$\frac{m\ddot{x}}{T} = \frac{-sin\alpha}{k(1-sin\alpha)} = sin\alpha - 1$

$\Rightarrow k = \frac{sin\alpha}{(1-sin\alpha)^2}$, as required

2nd part



As seen in the diagram, $y = \frac{x}{(1-x)^2}$ can take any value $k > 0$ when $0 < x < 1$, and we can set $x = sin\alpha$, where α is an acute angle.

(iii) From the 1st part of (i), $Tcos\alpha - kmg = km\ddot{x}cos\alpha$,

and from the 2nd part of (i), $Tsin\alpha - T = m\ddot{x}$

Hence $\frac{T}{m} = \frac{kg+k\ddot{x}cos\alpha}{cos\alpha} = \frac{\ddot{x}}{sin\alpha-1}$

$\Rightarrow kg(sin\alpha - 1) + \ddot{x}kcos\alpha(sin\alpha - 1) = \ddot{x}cos\alpha$

$\Rightarrow \ddot{x}cos\alpha\{k(sin\alpha - 1) - 1\} = kg(1 - sin\alpha)$

$\Rightarrow \ddot{x}cos\alpha = \frac{kg(1-sin\alpha)}{k(sin\alpha-1)-1} = \frac{kg(1-sin\alpha)^2}{k(sin\alpha-1)(1-sin\alpha)-(1-sin\alpha)}$

$= \frac{gsin\alpha}{-sin\alpha-1+sin\alpha} = -gsin\alpha$, so that $\ddot{x} = -gtan\alpha$, as required.