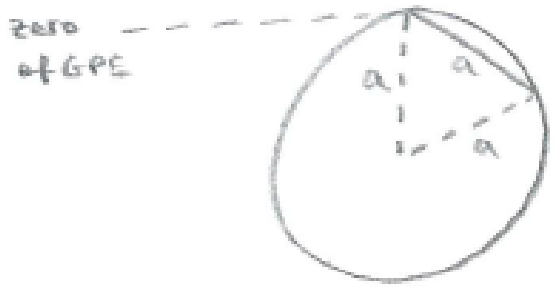
STEP 2012, Paper 3, Q10 – Solution (2 pages; 16/7/18)

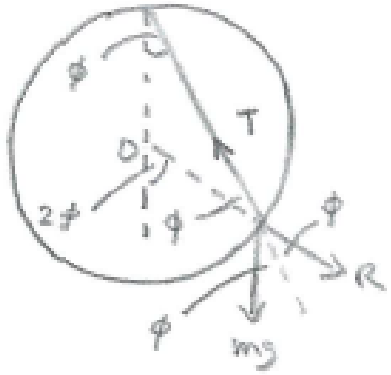


Taking the top of the hoop as the zero of GPE,

conservation of energy $\Rightarrow GPE + EPE + KE = \text{constant}$

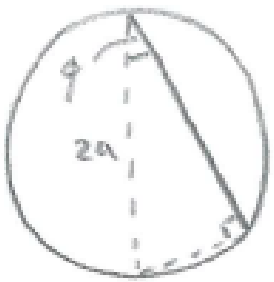
$\Rightarrow -mg(2a) + \frac{1}{2}\frac{\lambda}{a}a^2 + 0 = -mg(acos60°) + 0 + 0 \quad (1)$

$\Rightarrow \frac{\lambda a}{2} = mga(2 - 0.5) \Rightarrow \lambda = 3mg$



Circular motion $\Rightarrow T cos\phi - R - mgcos(2\phi) = \frac{mv^2}{a} \quad (2)$

[symbols need to be defined]



As can be seen from the above diagram, the stretched length of the string is $2acos\phi$.

From (1), conservation of energy $\Rightarrow -mg(acos60°)$

$= -mga(1 + cos2\phi) + \frac{1}{2}\frac{\lambda}{a}(2acos\phi - a)^2 + \frac{1}{2}mv^2$ (3)

Also, by Hooke's law, $T = \frac{\lambda}{a}(2acos\phi - a)$ (4)

From (2) & (4), $R = \frac{\lambda}{a}(2acos\phi - a)cos\phi - mgcos(2\phi) - \frac{mv^2}{a}$

And from (3), $\frac{mv^2}{a} = -mg + 2mg + 2mgcos2\phi - \lambda(2cos\phi - 1)^2$

So $R = 3mg(2cos\phi - 1)cos\phi - mgcos(2\phi)$

$-mg - 2mgcos2\phi + 3mg(2cos\phi - 1)^2$

$mg\{6cos^2\phi - 3cos\phi - 3cos2\phi + 12cos^2\phi - 12cos\phi + 2\}$

$= mg\{18cos^2\phi - 15cos\phi - 3(2cos^2\phi - 1) + 2\}$

$= mg\{12cos^2\phi - 15cos\phi + 5\}$, as required

Then, as $\Delta = 225 - 240 < 0$,

$12cos^2\phi - 15cos\phi + 5 > 0 \ \ \forall\phi$ [for all ϕ]

and so R is non-zero throughout the motion