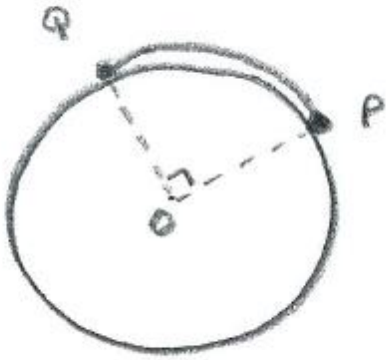
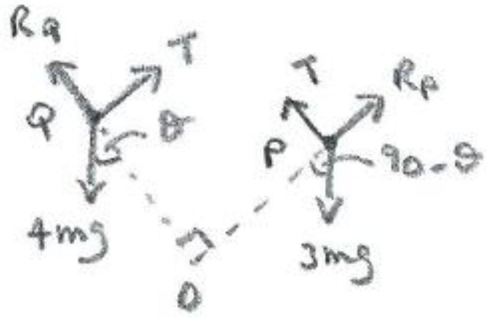
STEP 2011, Paper 3, Q9 – Solution (2 pages; 12/6/18)





Taking the zero of potential energy as the horizontal through O, by conservation of energy:

Initial PE = Final PE + Final KE

so that $(4m)g(acos\theta_0) + (3m)g(\text{acos}[90 - \theta_0])$

$= (4m)g(acos\theta) + (3m)g(\text{acos}[90 - \theta]) + \frac{1}{2}(7m)(a\dot{\theta})^2$ (1),

where $\theta = \theta_0$ in the initial equilibrium position.

When in equilibrium,

$T = 4mgsin\theta_0$ & $T = 3mgsin(90 - \theta_0) = 3mgcos\theta_0$

so that $4sin\theta_0 = 3cos\theta_0$

and $16sin^2\theta_0 = 9(1 - sin^2\theta_0)$

$\Rightarrow 25sin^2\theta_0 = 9$

$\Rightarrow sin\theta_0 = \frac{3}{5}$ (as $0 < \theta_0 < \pi, so\ that\ sin\theta_0 > 0$)

and $cos\theta_0 = \frac{4}{5}$ ($cos\theta_0 > 0$, as $0 < \theta_0 < \frac{\pi}{2}$; since if $\theta_0 = \frac{\pi}{2}$, P would be vertically above O, and equilibrium wouldn't be possible)

Then multiplying (1) by $\frac{2}{ma} \Rightarrow$

$8gcos\theta_0 + 6gsin\theta_0 = 8gcos\theta + 6gsin\theta + 7a\dot{\theta}^2$,

so that $7a\dot{\theta}^2 + 8gcos\theta + 6gsin\theta = 8g\left(\frac{4}{5}\right) + 6g\left(\frac{3}{5}\right) = 10g$, (2)

as required.

(i) The circular motion equation for Q is:

$(4m)a\dot{\theta}^2 = 4mgcos\theta - R_Q$

Q loses contact when $R_Q = 0$,

so that, as (2) still applies at this moment,

$10g - 8gcos\beta - 6gsin\beta = 7a\dot{\theta}^2 = 7gcos\beta$

$\Rightarrow 15cos\beta + 6sin\beta = 10$, as required.

(ii) Let the acceleration of P & Q be f

For Q: $4mgsin\theta - T = 4mf$ (3)

and for P: $T - 3mgcos\theta = 3mf$ (4)

Then, $4 \times (4) - 3 \times (3) \Rightarrow 7T - 12mgcos\theta - 12mgsin\theta$

$\Rightarrow T = \frac{12}{7}mg(sin\theta + cos\theta)$, as required.