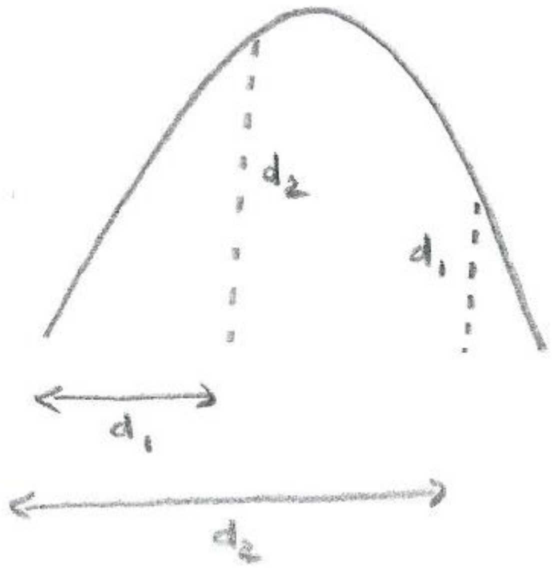
STEP 2011, Paper 1, Q9 – Solution (2 pages; 11/6/18)



If u is the initial speed of the particle, then (taking the point of projection as the Origin),

$x = ucos\theta.t$ & $y = usin\theta.t - \frac{1}{2}gt^2$

$\Rightarrow y = xtan\theta - \frac{gx^2}{2u^2cos^2\theta}$ (1)

Let $A = \frac{g}{2u^2}$

(1) $\Rightarrow$ $d_2 = d_1tan\theta - A{d_1}^2(1 + tan^2\theta)$ (2)

and $d_1 = d_2tan\theta - A{d_2}^2(1 + tan^2\theta)$ (3)

Then (2) $\Rightarrow$ $A = \frac{d_1tan\theta - d_2}{{d_1}^2(1+tan^2\theta)}$ (4) and (3) $\Rightarrow$ $A = \frac{d_2tan\theta - d_1}{{d_2}^2(1+tan^2\theta)}$

Equating these two expressions,

$(d_1tan\theta - d_2){d_2}^2 = (d_2tan\theta - d_1){d_1}^2$

$\Rightarrow tan\theta(d_1{d_2}^2 - d_2{d_1}^2) = {d_2}^3 - {d_1}^3$

$\Rightarrow tan\theta = \frac{(d_2-d_1)({d_1}^2+d_1d_2+{d_2}^2)}{d_1d_2(d_2-d_1)} = \frac{{d_1}^2+d_1d_2+{d_2}^2}{d_1d_2}$, as required (5)

From (1), the range R satisfies $Rtan\theta - AR^2(1+tan^2\theta) = 0$

Then, as $R \neq 0$, $tan\theta - AR(1+tan^2\theta) = 0$,

so that $R = \frac{tan\theta}{A(1+tan^2\theta)}$

From (4), $A(1+tan^2\theta) = \frac{d_1tan\theta - d_2}{{d_1}^2}$,

so that, from (5), $R = \frac{tan\theta.{d_1}^2}{d_1tan\theta-d_2} = \frac{({d_1}^2+d_1d_2+{d_2}^2)\left(\frac{d_1}{d_2}\right)}{\left(\frac{{d_1}^2+d_1d_2+{d_2}^2}{d_2}\right)-d_2}$

$= \frac{d_1({d_1}^2+d_1d_2+{d_2}^2)}{{d_1}^2+d_1d_2+{d_2}^2-{d_2}^2} = \frac{{d_1}^2+d_1d_2+{d_2}^2}{d_1+d_2}$