CAMBRIDGE ASSESSMENT

STEP Examiners' Report 2008

Mathematics

STEP 9465, 9470, 9475

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

Contents

Step Mathematics (9465, 9470, 9475)

General Remarks

There were significantly more candidates attempting this paper this year (an increase of nearly 25%), but many found it to be very difficult and only achieved low scores. The mean score was significantly lower than last year, although a similar number of candidates achieved very high marks. This may be, in part, due to the phenomenon of students in the Lower Sixth (Year 12) being entered for the examination before attempting papers II and III in the Upper Sixth. This is a questionable practice, as while students have enough technical knowledge to answer the STEP I questions at this stage, they often still lack the mathematical maturity to be able to apply their knowledge to these challenging problems.

Again, a key difficulty experienced by most candidates was a lack of the algebraic skill required by the questions. At this level, the fluent, confident and correct handling of mathematical symbols (and numbers) is necessary and is expected; many students were simply unable to progress on some questions because they did not know how to handle the algebra.

There were of course some excellent scripts, full of logical clarity and perceptive insight. It was also pleasing that one of the applied questions, question 13, attracted a very large number of attempts.

However, the examiners were again left with the overall feeling that some candidates had not prepared themselves well for the examination. The use of past papers and other available resources to ensure adequate preparation is strongly recommended. A student's first exposure to STEP questions can be a daunting, demanding experience; it is a shame if that takes place during a public examination on which so much rides.

Comments on individual questions

Q1 This question was primarily about logical thinking and structuring an argument. While it was a very popular question, the marks were disappointing: only 30% of candidates gained more than 6 marks.

Most candidates could describe vaguely what is meant by the term irrational, though only a handful gave a precise, accurate definition. The popular offering of 'a number with an infinite decimal expansion' was not acceptable.

It was pleasing to see, though, that the majority of candidates were capable of using proof by contradiction to prove statements A and B, and they then went on to provide a counterexample to statement C. A small number of very strong candidates justified their counterexamples by proving that the numbers they presented were in fact irrational, though any well-known irrational examples were given full marks without the need for justification.

It is important to stress the difference between proving a statement and disproving one; while a single (numerical) counterexample is adequate to disprove a statement, a proof of the truth of a statement requires a general argument. Too many candidates wrote things such as: 'If $pq = \sqrt{3}$, then $p = \sqrt{3}$ and $q = 1$, so . . . .' Also, it is unknown what an irrational number 'looks like', so the frequently occurring arguments such as 'We know that e + π must be irrational because the numbers are not of the same form' (when comparing this example to something like $(1 - \sqrt{2}) + \sqrt{2}$) are spurious.

Sadly, very few candidates made any significant progress on the main part of the question. Several attempted (unsuccessfully) to prove that all four of the given numbers are irrational. Others asserted that since π and e are both irrational, π + e must also be, despite having just disproved statement C. A number of candidates successfully showed that π + e and π - e cannot both be irrational by appealing to B, but then could not see how to continue. The best attempts proceeded by using A and B repeatedly to show that no pair of $\pi \pm$ e and $\pi^2 \pm e^2$ could simultaneously be rational (that is, they considered all six cases separately).

Q2 This was by far the most popular question on the paper, with about six out of every seven candidates attempting it.

The very first part involving implicit differentiation was generally done very well with most candidates scoring full marks for this part.

A majority of candidates then went on to successfully see how to apply this result to the required integral, although a sizeable minority failed to understand that they were being asked to perform a substitution. Some candidates resorted to the formula book and quoted the standard integral $\int 1/\sqrt{x^2 + a^2}\,\mathrm{d}x$; however, this gained no credit as the question explicitly said 'hence'.

Having reached $\int 1/(t+b)\,\mathrm{d}t$, the vast majority of candidates became unstuck. Firstly, after integrating, some did not substitute back $t = x + \cdots$ to get an expression in terms of x. The fundamental problem, though, was that the candidates were mostly unaware of the need to use absolute values when integrating $1/x$: almost everyone gave the intermediate answer as $\ln(t + b) + c$ rather than $\ln|t + b| + c$. It turns out that in this case, $t + b$ is always positive so the absolute values may be replaced by parentheses, but this requires explicit justification (which no-one gave).

This lack of appreciation of absolute values prevented all but the strongest candidates from making a decent attempt at the last part of the question, the consideration of the case $c = b^2$. Some candidates successfully substituted this in to the earlier result as instructed, but many claimed that $\sqrt{2x^2 + 2x + b^2} = x + b$. However, the correct expression is $|x + b|$, which is $x + b$ when this is positive, but $-(x + b)$ when $x + b < 0$. Only the tiny handful of candidates who appreciated this subtlety managed to correctly explain the distinction between these two cases.

Q3 This was another popular question, although the scores were again fairly poor.

The proof of (*) was often done quite well. The main difficulties here arose because of a lack of clarity in the logic; it is important to make clear where the starting point is and what steps are being taken to move forward from there. A significant number of candidates attempted to work backwards, and then divided by $d - b$ or the like without realising that this might be zero. Also, inequalities were multiplied without any regard to the sign of the numbers under consideration; for example, while $0 > -1$ and $1 > -2$, it is not true that $0 \times 1 > (-1) \times (-2)$.

The beginning of part (i) was completed correctly by a majority of candidates. It is important to stress again that if a question specifies "use (*)", then this must be done to gain any credit; no marks were given for the numerous answers which began with "as $(x - y)^2 \geq 0$ for all x and y, we have $x^2 - 2xy + y^2 \geq 0$" or similar.

The last part of (i) was often poorly tackled. It was sometimes interpreted to mean "when $x < 0$" or other spurious cases, without understanding that the inequality had so far only been shown in the case $x \geq y \geq z$. (Indeed, the intermediate result $z^2 + xy \geq xz + yz$ does not hold in the case $x > z > y$.) Many other candidates ignored their preceding work and went on to prove the result from scratch using the inequality $(x - y)^2 + (y - z)^2 + (z - x)^2 \geq 0$. Very few candidates explained the symmetry of the situation.

Part (ii) was problematic because of the wording of the question. It turned out that there is a very straightforward way to answer this part by making use of the results proved in part (i). While this was not what was actually asked ("Show similarly . . . "), it was felt unfair to penalise candidates too harshly for taking this route. Thus they were awarded partial credit and **all** such candidates were referred to the Chief Examiner for individual consideration. Nonetheless, the attempts at this part, by whichever method, were generally either close to perfect or non-starters.

Q4 The initial graph-sketching part of this question was designed to help candidates solve the quadratic equation which was to come up later in the question. Whilst almost all of the candidates successfully sketched $y = \sin x$, the attempts at $y = \frac{2}{3}\cos^2 x$ were significantly poorer. Many candidates sketched curves with cusps at the x-axis, presumably confusing $\cos^2 x$ with $|\cos x|$; others had curves which fell below the x-axis in places. Perhaps few candidates had seen graphs of $y = \cos^2 x$ before or considered that $\cos^2 x = \frac{1}{2}(\cos 2x + 1)$, making $\cos^2 x$ sinusoidal itself. Also, a large number of candidates appeared to have spent a significant amount of time drawing beautiful and accurate graphs on graph paper; it is important to appreciate the nature of a sketch as a rough drawing which captures the essential features of a situation. In general, STEP questions will not require accurate graphs, only accurate sketches.

The first derivative of $f(x)$ was generally computed correctly, though a sizable proportion of candidates failed to correctly apply the product rule to determine the second derivative. Those candidates who obtained $f''(x)$ correctly generally realised that they needed to solve the inequality $\frac{2}{3}\cos^2 x \geq \sin x$. Some appear to have guessed a value of x which makes this an equality - this method is perfectly acceptable as long as some justification of the claimed result is given (such as by explicitly substituting $x = \pi / 6$ into the two sides). Most candidates who got this far correctly understood the connection with the graph sketch and went on to give the correct intervals.

In part (ii), there was a lot of difficulty performing the differentiation. A number of candidates made their life more difficult by substituting $k = \sin 2\alpha$ before differentiating $g(x)$; this just made the expressions appear more complex and increased the likelihood of error. Some candidates, for example, tried differentiating with respect to both x and α simultaneously.

Nevertheless, most candidates who were able to correctly compute g''(x) went on to solve the resulting trigonometric equation, finding the solution $x = \alpha$, but many failed to determine the second interval. A sketch of some sort would very likely have been useful.

Q5 This was the least popular of the Pure Mathematics questions. There was a fair amount of confusion as to the meaning of the summation, with the majority of attempts at the n =1 case in part (i) failing to understand that the polynomial would be $p(x)= x+a_0$ rather than just $p(x)= x$. A small number thought that the summation indicated a geometric series, and proceeded to claim that $p(x) = (1 - x^n)/(1 - x)$ or other such things.

Nonetheless, there were many good answers to the rest of part (i), with candidates showing that they understood the statement of Chebyshev's theorem. A small number of strong candidates had a mature enough understanding of mathematics to use the alternative method given in the sample solutions; most were content with finding the maxima and minima. Some forgot to check the value of p(x) at the ends of the interval, which was not penalised as long as they did not incorrectly assert that they had found the value of *M*.

One recurrent incorrect assertion was that from the inequality $p(x) \geq \frac{1}{2}$, it necessarily follows that $M = \frac{1}{2}$, without showing that equality is obtained for *some* value of x.

There were few serious attempts at part (ii), but most of those achieved full marks or very close to it. Several candidates had difficulty in explaining their reasoning: a sketch would certainly have helped clarify why a maximum value of $|p(x)|$ occurring in the interval $-1 < x < 1$ necessarily forces this point to be a turning point.

Of the other attempts, many could not see the relevance of Chebyshev's theorem to this situation, or even if they did, then failed to divide the given polynomial by 64. Arguments which did not invoke Chebyshev's theorem were not given any credit (the main alternative being to differentiate, then to find points where the derivative was positive and negative, and use the intermediate value theorem to assert that there is a point where the derivative must be zero).

Q6 This was another popular question which was gained a pleasing number of good marks.

The sketch was generally done well. A significant number of candidates did not realised that f(0) = 0 and f(1) = 1, so either had non-intersecting graphs or graphs which were tangent to each other at the origin. A number of candidates sketched the graph of f(x) for all real x, in spite of the question stating $x \geq 0$; they were not penalised for this. Most understood how the graphs of f(x) and g(x) related.

The determination of g(x) algebraically was performed correctly by a majority of candidates. However, a disturbing number of candidates introduced absolute value signs, writing $g(x)= \ln |(e-1)x+1|$. Whilst technically correct in this range (and therefore not penalised here), it is indicative of confusion about when absolute values are used

with logs: when integrating $1/x$ (as in Question 2 above) they are required; when inverting exponentiation they are not. A smaller number made very significant errors in their handling of the logarithm function, writing such things as
$\ln(ex - x + 1) = \ln ex - \ln x + \ln 1$.

The majority of candidates correctly integrated $f(x)$. A small minority bizarrely asserted that $\int_0^{1/2} f(x)dx = f\left(\frac{1}{2}\right) - f(0)$ which was somewhat disturbing.

The integration of $g(x)$ proved much more troublesome. Despite $\int \ln x \, dx$ being a standard integral and explicitly mentioned in both the STEP Specification and A2 Mathematics specifications, the introduction of the linear function of x flummoxed most candidates. Some differentiated instead of integrating, others just gave up. A small number either attempted to use parts or to substitute, and a good proportion of such attempts were successful. Some candidates confused differentiation with integration during this process and tried to use a mixture of parts and the product rule.

Finally, of those who managed to reach this point, a decent number gave a very convincing explanation of why $\int f + \int g = \frac{1}{2}k$.

Q7 This was a reasonably popular question, tackled by about half of the candidates. Most confidently showed that $y = \frac{1}{2}(y - \sqrt{3}\,x)$ and went on to deduce the result for the clockwise rotation. A small number of candidates lost marks here because their presentation either failed to make clear which answer corresponded to which direction of rotation, or the directions were reversed. Several candidates would have been helped by including a sketch in their solution.

About two-thirds of the candidates were unable to progress beyond this point. Of those who continued, the majority succeeded in finding h_1, either by a direct argument or, more usually, by using the earlier result as intended by the question. Despite the hint of h_1 being given with absolute value signs, a large number of candidates then claimed that $h_2 = y$ rather than the correct $|y|$, suggesting that they do not understand what absolute values mean and when they should be used.

Very few candidates correctly determined h_3, the most common incorrect answer being $h_3 = \frac{1}{2}|y + \sqrt{3}\,x|$ to parallel the answer for h_1. Again, clear diagrams are essential if marks are to be gained for questions such as these. There was also evidence of confusion in the algebraic manipulation of absolute values, with some candidates confusing $|a-b|$ with $|a|-|b|$, thereby giving answers such as $h_3 = \frac{1}{2}|y + \sqrt{3}\,x| - \frac{1}{2}\sqrt{3}$

Only a handful of candidates made a significant attempt at the final part of the question, and of those who did, the main difficulty stemmed from not appreciating that to prove an “if and only if” statement, one has to prove the implication in both directions. The sample solutions use the triangle inequality; it could equally and straightforwardly be argued by considering all eight possible cases of where the point P might lie with respect to each of the three sides of the triangle.

Q8 This was another popular question, and many candidates achieved decent marks on this question.

Many candidates were correctly able to differentiate (*), although a significant number ran into difficulties with the $(y')^2$ term, where things such as $2y'\frac{\mathrm{d}}{\mathrm{d}x}(y') = 2y'.y''\frac{\mathrm{d}y}{\mathrm{d}x}$ were common errors. Although the rest of (*) was usually differentiated correctly by these candidates, since the rest of part (i) depended upon getting this first step correct, they floundered from then on. Many of these candidates nevertheless went on to gain additional marks by at least making a good start to part (ii).

Also, candidates must remember to read the question and to follow its guidance; the question instructed them to differentiate (*), and those who tried rearranging it instead got nowhere.

From $y'' = 0$, most candidates deduced that $y' = \mathrm{m}$ and substituted this back into (*) to determine $y = mx - m^2$. However, when working like this, it is vital to check that the purported y, call it $\hat{y}$ say, satisfies $\mathrm{d}\hat{y}/\mathrm{d}x = y'$, since y and y' must be related by both the given differential equation and also by $y' = \mathrm{d}y/\mathrm{d}x$. There may be other arguments which would allow one not to differentiate the obtained y, but these would have to be given explicitly. The alternative method of determining that $y = mx + c$ and then substituting this into (*) was noticeably less common, but avoided this subtlety.

For the $2y'' = x$ case, similar comments again apply, although here it was concerning how many candidates integrated to get y = $\frac{1}{4}x^2$ without including a constant of integration.

In part (ii), few candidates succeeded in correctly differentiating the differential equation. One of the most common errors was to claim that $\frac{\mathrm{d}}{\mathrm{d}x}(y^2) = 2y'$. The few candidates who correctly differentiated the equation mostly applied the techniques from part (i) to solve the equation successfully. Several fudged the solution of the resulting equation $(x^2 - 1)\ y' = xy$ by conveniently forgetting the absolute value signs when integrating (as shown in the sample solutions), but this error was not penalised on this occasion.

Q9 This was an unpopular question and the marks were very disappointing; only half of the attempts gained over one mark out of twenty, and only a handful of candidates gained over six marks.

Nonetheless, of the candidates who made a reasonable start, many were capable of drawing a clear diagram of the position of the hoop after it had rolled, but few were able to show how the position after it had rolled related to its initial position. This allowed them to correctly determine the y-coordinate of P, but they became very unstuck when attempting to determine the x-coordinate.

The next difficulty encountered was in calculating the components of the velocity of P, as many candidates appeared unable to differentiate a function of θ with respect to t.

For the determination of the kinetic energy, several candidates $\dot{\theta}$ used the expected method. It was also very encouraging to see a number of candidates correctly using the formula total KE = linear KE + rotational KE, and then determining the rotational KE using either moments of inertia or the explicit formula $\frac{1}{2}mr^2\dot{\theta}^2$.

Finally, a small number of candidates correctly considered forces or energy and deduced that the hoop rolls at constant speed.

Q10 This was the most popular mechanics question, and the question which gained the best marks across the entire paper.

The sketch of the particle's trajectories in the two different scenarios was generally well done, with almost all candidates successfully completing the sketch. It was a little disappointing, though, that very few attempted to justify their assumption that the particle does, in fact, reach height h.

The next stage, using the "suvat" equations to deduce d, was generally either done very well or very poorly. Of those who had difficulty, some were stuck trying to figure out how to go about the question, others were unsure of which of the "suvat" equations to use (despite all of the individual components of this question being very standard A-level problems), while some derived a quadratic equation (having used $s = ut + \frac{1}{2}at^2$) but were incapable of then solving it.

Nonetheless, this question did require a sustained chain of logical steps, and it was pleasing to see over a quarter of the candidates who attempted this question gaining close to full marks on it.

Q11 This was the least popular of the mechanics questions, and of the candidates who attempted it, only a handful made any progress beyond drawing a usually incorrect sketch and writing down some equations.

The majority were aware that $F = \mu R$ as the equilibrium is limiting. Unfortunately, though, they often either missed forces from their diagram or drew at least one of the frictional forces in the wrong direction. Another frequent problem was that they labelled both normal reaction forces with the same variable R, thereby implicitly implying that they are equal, whereas this is not the case. A small annoyance was the number of candidates who used the notation Fr for friction; an unhealthy practice as it can so easily be confused with $F \times r$. Also, several failed to mark the angle α correctly on their diagram.

After this, a small number of candidates correctly resolved in two directions and took moments. Those who understood how to then manipulate the resulting equations to eliminate most of the variables went on to produce essentially perfect solutions, whereas everyone else became stuck at this point and found themselves unable to progress any further.

No attempts using the theorem regarding three forces on a large body were seen, which is a shame, as it made the problem significantly easier.

Q12 This was by far the least popular question on the paper, as is often the case with Probability and Statistics questions.

Of the candidates who attempted it, most successfully answered part (i), and a significant number were also confident in the manipulation of sums required for part (ii). Success was clearly dependent upon taking great care to ascertain the meaning of the event $X = r$ in terms of X_1 and X_2.

Part (iii) proved much more problematic, as almost no-one made use of *both* defining inequalities for the median; one inequality on its own may appear to give the correct answer, but is insufficient to gain the marks.

The handful of candidates who attempted part (iv) were generally successful in their attempts.

Q13 This combinatorics question was attempted by close to half of all candidates, a very encouraging statistic. About two-thirds of the attempts did not progress very far, gaining five marks or fewer, but of those who did get further, the marks were fairly evenly distributed.

For part (i), most attempts reached the stated answer, although a significant number used very creative, if inaccurate or meaningless, ways of doing so. The majority of candidates used counting methods, and many of these were successful to a greater or lesser extent. The other method used by many candidates was to consider the probability of the first wife sitting next to her husband ($\frac{2}{5}$) and the conditional probability of the spouse of the other person sitting next to the first husband sitting next to them (this is $\frac{1}{3}$), and then multiplying these.

It is crucial at this point to reinforce that candidates must explain their reasoning in their answers, especially when they are working towards a given answer. Simply writing $\frac{2}{5} \times 13 = \frac{2}{15}$ is woefully inadequate to gain all of the available marks; there must be a justification of the reasoning behind it.

Parts (ii) and (iii) were found to be a lot more challenging. A number of candidates attempted to construct probabilistic arguments, which are very challenging in this case. The successful attempts all used pure counting arguments. The examiners often found it challenging to decipher their thinking, though, as the explanations were often somewhat incoherent. Those who used counting arguments usually made good progress on both parts.

The favoured method for part (iii) was to use P(no pairs) = 1 − P(≥ 1 pair). It would have certainly been worth checking the answer obtained using a direct method, as this would have caught a number of errors.

The main errors encountered in good attempts at the later parts of the question were a failure to consider all possible cases or a miscounting of the number of ways each possible case could occur.

Overall, this question was answered well by a significant number of candidates.

General Remarks

There were around 850 candidates for this paper – a slight increase on the 800 of the past two years – and the scripts received covered the full range of marks (and beyond!). The questions on this paper in recent years have been designed to be a little more accessible to all top A-level students, and this has been reflected in the numbers of candidates making good attempts at more than just a couple of questions, in the numbers making decent stabs at the six questions required by the rubric, and in the total scores achieved by candidates. Most candidates made attempts at five or more questions, and most genuinely able mathematicians would have found the experience a positive one in some measure at least. With this greater emphasis on accessibility, it is more important than ever that candidates produce really strong, essentially-complete efforts to at least four questions. Around half marks are required in order to be competing for a grade 2, and around 70 for a grade 1.

The range of abilities on show was still quite wide. Just over 100 candidates failed to score a total mark of at least 30, with a further 100 failing to reach a total of 40. At the other end of the scale, more than 70 candidates scored a mark in excess of 100, and there were several who produced completely (or nearly so) successful attempts at more than six questions; if more than six questions had been permitted to contribute towards their paper totals, they would have comfortably exceeded the maximum mark of 120. While on the issue of the "best-six question-scores count" rubric, almost a third of candidates produced efforts at more than six questions, and this is generally a policy not to be encouraged. In most such cases, the seventh, eighth, or even ninth, question-efforts were very low scoring and little more than a waste of time for the candidates concerned. Having said that, it was clear that, in many of these cases, these partial attempts represented an abandonment of a question after a brief start, with the candidates presumably having decided that they were unlikely to make much successful further progress on it, and this is a much better employment of resources.

As in recent years, most candidates' contributing question-scores came exclusively from attempts at the pure maths questions in Section A. Attempts at the mechanics and statistics questions were very much more of a rarity, although more (and better) attempts were seen at these than in other recent papers.

Comments on individual questions

Q1 The first question is invariably intended to be a gentle introduction to the paper, and to allow all candidates to gain some marks without making great demands on either memory or technical skills. As such, most candidates traditionally tend to begin with question 1, and this proved to be the case here. Almost 700 candidates attempted this question, making it (marginally) the second most popular question on the paper; and it gained the highest mean score of about 14 marks.

There were still several places where marks were commonly lost. In (i), setting $(x_2, y_2) = (x_1, y_1)$ and eliminating y (for instance) leads to a quartic equation in x. There were two straightforward linear factors easily found to the quartic expression, leaving a quadratic factor which could yield no real roots. Many candidates failed to explain why, or show that, this was so. In (ii), the algebra again leads to two solutions, gained by setting $(x_3, y_3) = (x_1, y_1)$. However, one of them corresponds to one of the solutions already found in (i), where the sequence is constant, and most candidates omitted either to notice this or to discover it by checking. Another very common oversight – although far less important in the sense that candidates could still gain all the marks by going the long way round – was that the algebra in (ii) was *exactly the same* as that in (i), but with $a = -x$ and $b = -y$. For the very few who noticed this, the working for the second half of the question was remarkably swift.

Q2 Noticeably less popular than Q1 – with only around 500 "hits" – and with a very much poorer mean mark of about 8, it was rather obvious that many candidates were very unsure as to what constituted the *best* partial fraction form for the given algebraic fraction to begin with. Then, with very little direct guidance being given in the question, candidates' confidence seemed to ebb visibly as they proceeded, being required to turn the resulting collection of single algebraic fractions into series, using the *Binomial Theorem*, and then into a consideration of general terms. There was much fudging of these general terms in order to get the given answers of either $n + 1$ or $n + 2$ for the general term's coefficients; even amongst those who did spot which one occurred when, there was often little visible justification to support the conclusions. As a result of all the hurdles to be cleared, those who managed to get to the numerical ending successfully were very few in number.

Q3 This, the third most popular question on the paper, producing a mixed bag of responses. It strikes me that, although the A-level specifications require candidates to understand the process of *proof by contradiction*, this is never actually tested anywhere by any of the exam. boards. Nonetheless, it was very pleasing to see that so many candidates were able to grasp the basic idea of what to do, and many did so very successfully. The impartial observer might well note that the situation in (i) is very much tougher (in terms of degree) than that in (ii). However, candidates were very much more closely guided in (i) and then left to make their own way in (ii).

Apart from the standard, expected response to (i) – see the *SOLUTIONS* document for this – many other candidates produced a very pleasing alternative which they often dressed up as *proof by contradiction* but which was, in fact, a direct proof. It was, however, so mathematically sound and appealing an argument (and a legitimate imitation of a *p by c*) that we gave it all but one of the marks available in this part of the question. It ran like this:

> Suppose w.l.o.g. that $0 < a \le b \le c < 1$.
> Then $ab(1 - c) \le b^2(1 - b) \le \frac{4}{27}$ by the previous result
> (namely $x^2(1 - x) \le \frac{4}{27}$ for all $x \ge 0$).
> QED.

[Note that we could have used $ab(1 - c) \le c^2(1 - c) \le \frac{4}{27}$ also.]

It has to be said that most other inequality arguments were rather poorly constructed and unconvincing, leaving the markers with little option but to put a line through (often) several pages of circular arguments, faulty assumptions, dubious conclusions, and occasionally correct statements with either no supporting reasoning or going nowhere useful.

There was one remarkable alternative which was produced by just a couple of candidates (that I know of) and is not included in the *SOLUTIONS* because it is such a rarity. However, for those who know of the *AM – GM Inequality*, it is sufficiently appealing to include it here for novelty value. It ran like this:

Assume that $bc(1 - a),\ ca(1 - b),\ ab(1 - c) > \frac{4}{27}$.
Using the previous result, we have $a^2(1 - a),\ b^2(1 - b),\ c^2(1 - c) \le \frac{4}{27}$.
Then, since all terms are positive, it follows that $a^2 \le bc,\ b^2 \le ca,\ c^2 \le ab$ so that

$$a^2 + b^2 + c^2 \le bc + ca + ab. \textbf{(*)}$$

However, by the *AM – GM Inequality* (or directly by the *Cauchy-Schwarz Inequality*),

$$a^2 + b^2 \ge 2ab\ ,\ \ b^2 + c^2 \ge 2bc \ \text{ and } \ c^2 + a^2 \ge 2ca\ .$$

Adding and dividing by two then gives $a^2 + b^2 + c^2 \ge ab + bc + ca$, which contradicts the conclusion **(*)**, etc., etc.

Q4 Another very popular question, poorly done (600 attempts, mean score below 7). Most efforts got little further than finding the gradient of the normal to the curve, and I strongly suspect that this question was frequently to be found amongst candidates' non-contributing scorers. Using the tan ($A - B$) formula is a sufficiently common occurrence on past papers that there is little excuse for well-prepared candidates not to recognise when and how to apply it. Once that has been done, the question's careful structuring guided able candidates over the hurdles one at a time, each result relying on the preceding result(s); yet most attempts had finished quite early on, and the majority of candidates failed to benefit from the setters' kindness.

Q5 This was the most popular question on the paper (by a small margin) and with the second highest mean mark (12) of all the pure questions. Those who were able to spot the two standard trig. substitutions $s = \sin x$ and $c = \cos x$ for the first two parts generally made excellent progress, although the log. and surd work required to tidy up the second integral's answer left many with a correct answer that wasn't easy to do anything much useful with at the very end, when deciding which was numerically the greater. The binomial expansion of $(a + b)^5$ was handled very comfortably, as was much of the following inequality work. However, the very final conclusion was very seldom successfully handled as any little mistakes, unhelpful forms of answers, etc., prevented candidates' final thoughts from being sufficiently relevant.

Q6 This was the least popular of the pure maths questions. Although there were 300 starts to the question, most of these barely got into the very opening part before the attempt was abandoned in favour of another question. Most attempts failed to show that $f(x)$ has a period of 4π. As mentioned, few proceeded further. Of those who did, efforts were generally very poor indeed – as testified to by the very low mean mark of 4 – with the necessary comfort in handling even the most basic of trig. identities being very conspicuous by its absence. Part (iii) was my personal favourite amongst the pure questions, as it contained a very uncommon – yet remarkably simple – idea in order to get started on the road to a solution. The idea is simply this: $f(x)$, being the sum of a cosine term and sine term, is equal to 2 *if and only if* each of these separate terms is simultaneously at its maximum of 1. That is, the question is actually two very easy trig. equations disguised as one very complicated-looking one. Once realised, the whole thing becomes very straightforward indeed, but only a few candidates had persevered this far.

Q7 In many ways, part (i) of the question was very routine, requiring little more than technical competence to see the differential equation, using the given substitution, through to a correct solution. Part (ii) then required candidates to spot a slightly different substitution on the basis of having gained a feel for what had gone on previously. I had thought that many more candidates would try something involving the square root of $1 + x^3$ or the cube root of $1 + x^2$, rather than cube root of $1 + x^3$, but many solutions that I saw went straight for the right thing. Once this had been successfully pushed through – with the working mimicking that of (i) very closely indeed – it was not difficult to spot the general answer required, unproven, in (iii). Overall, however, it seems that a lot of candidates failed to spot the right thing for part (ii) and their solutions stopped at this point. With almost 600 attempts, the mean score on this question was 10.

Q8 As with Q6, this was both an unpopular question and poorly done. Those candidates who did do well generally did so after spotting that they could use the *Angle Bisector Theorem* to polish off the first half of the question, expressing λ in terms of *a* and *b* almost immediately. Predominantly, the whole thing relied almost exclusively upon the use of the scalar product (or, alternatively, the *Cosine Rule*) and a bit of manipulation. The fact that the mean mark on this question was below 7 is simply indicative of the general lack of confidence amongst candidates where vectors are concerned.

Q9 Of the applied maths questions, this was by far the most popular, with over 400 attempts. However, most of these were only partial efforts, with few candidates even getting around to completing part (i) successfully, and the mean score ended up at about 8. Most candidates were comfortable with the routine stuff to start with, quoting and using the trajectory equation and using the identity $\sec^2\alpha = 1 + \tan^2\alpha$ to get a quadratic equation in $\tan\alpha$. For the remaining parts of the question, working was much less certain, even given the helpful information about small-angle approximations, and very few candidates were able to get a suitable approximation for $\tan\alpha$. Fewer still could turn an angle in radians into one in degrees.

Q10 Though much less popular than Q9, the attempts at this question followed a similar pattern, with most candidates coping pretty well with the routine opening demands – the use of the two main principles governing collisions questions: *Conservation of Linear Momentum* and *Newton's Experimental Law of Restitution* – but then falling down when a little more care and imagination were required in the parts that followed. With some careful application of ideas relating to similar triangles and a bit of inequalities work to follow, most candidates attempting these questions were just not up to the task. Few got as far as working on the initial and final kinetic energies; of these only a very small number noticed that there was a very quick way to go about it (see the *SOLUTIONS*). I don't recall seeing anyone successfully managing to get the right answer after having taken the longer route.

Q11 This question attracted under 100 attempts and a mean mark of under 3. The strong complaint I have made in the Report over recent years has consistently been that candidates' efforts on such questions have been seriously compromised by a disturbing inability to draw a decent diagram at the outset. I'm afraid that this was a major stumbling-block to successful progress with this question this year also. It was also a bit of a problem that candidates tended to confuse the acceleration of *P* relative to the wedge with its absolute acceleration relative to the stationary surface on which the wedge stood (say). As few decent attempts were made, it is difficult to be very specific about what went on otherwise.

Q12 There were almost 200 attempts to Q12, and the mean score was the highest – at 14 – of all the applied questions. This was partly due to the fact that the result of the first part could be largely circumvented by anyone who knew a little bit about expectation algebra, enabling them to write down $E(X)$ straightaway. The simple combinations of events, and their associated probabilities, in the final part of the question were very confidently and competently handled by most candidates and many polished the question off in its entirety relatively quickly.

Q13 Perhaps encouraged by the ease with which they had managed Q12, many of these candidates went on to attempt this question also. Although the listing of relevant cases was a fairly straightforward exercise, the handling of the binomial coefficients – which certainly *looked* clumsy and unappealing – was coped with much less well, and many mistakes were made in the ensuing algebra. In the very final part of the question, the idea that the calculus could lead to a nice, neat answer $\left(k = \sqrt{n(n-1)}\right)$ that then needed to be interpreted in terms of integer values, was just one step too far for most takers. The eventual mean score of 8 on this question testifies to the difficulties found in the algebra by most of the candidates who attempted it.

General Remarks
Most candidates attempted five, six or seven questions, and scored the majority of their total score on their best three or four. Those attempting seven or more tended not to do well, pursuing no single solution far enough to earn substantial marks.

Comments on individual questions
Q1 This was the most popular question on the paper, and many earned good marks on it. Nearly all the candidates followed the hint, and most then applied the same trick with the third equation. Subsequent success depended on a candidate realising that they had simultaneous equations in xy and $x + y$, although very rarely some managed to solve directly in x and y.

Q2 About three fifths attempted this question, often obtaining the starred result and the familiar $S3(n)$ successfully, but with $S4(n)$ tripping up many. Any that made progress on part (ii) tended to be able to complete the whole question.

Q3 Just under half attempted this. Most were reluctant to use parametric differentiation. Some found *T*'s coordinates successfully and got not further, but most either made very little progress on the whole question, or got right through it.

Q4 Almost exactly the same number attempted this as question 3, but with much less success. The initial inequality was frequently poorly justified, but some managed to apply it correctly to obtain the starred result, and went on to do part (ii) respectably. However, for most, it was a case of all or nothing.

Q5 In terms of attempts and success, this resembled question 2. Apart from some that made no progress at all, the induction was accessible to many, as was the expression for Tn(x). In both of these there were frequent gaps or inaccuracies even though the solutions were understood in essence.

Q6 More than 80% attempted this, and with more success than any other question. Having obtained the relation between x and p in each part, quite a few attempts then treated these as differential equations rather than merely substituting back to find expressions for y, and consequent inaccuracies lost marks.

Q7 Less than a fifth attempted this and frequently with little success except for obtaining the initial result. The configuration for part (i) tripped up many, although some skipped that to do part (ii) successfully.

Q8 Three fifths attempted this with most scoring about two thirds of the marks. Apart from minor errors, the last part (expressing T in partial fractions etc.) was the pitfall for most.

Q9 Of the three Mechanics questions, this was the most popular with just under a quarter of the candidates attempting it, but with least success. In spite of obtaining the relation in the stem of the question, many failed to appreciate its consequence for the acceleration-time graph in part (i) and as a consequence made little further progress. If candidates managed part (i), then they tended to complete the question barring minor errors, and the occasional assumption that the final simple case was simple harmonic motion.

Q10 Just under a fifth attempted this, but many dealt successfully with the *n* short strings case to earn about half the marks. Occasionally a candidate would obtain the required length result for the heavy rope and fail to apply the same technique for the elastic energy, but apart from minor errors, most that appreciated how to take the limit had few difficulties.

Q11 Under a twelfth tried this. A number of different correct approaches were successfully applied, and there were very few partially correct solutions.

Q12 Little more than a handful of candidates attempted this with three strong attempts (near full marks) and the remainder making no headway at all.

Q13 About a ninth tried this. Apart from those who had no idea, there were three categories of attempt. The first group obtained the first result but did not spot that regardless of what happens in the first step, immediately after it there are $2n - 2$ free ends. The second group safely navigated the results for the general case but could not see how to apply the approximation to obtain the result in the specific case, and the final group had the satisfaction of finding the result. Most fell into the first category, with fewer in the second, and a small number in the third.

STEP Mathematics (9465/94709475)
June 2008 Assessment Series

Grade boundaries

Paper	Maximum Mark	S	1	2	3	U
Paper 1 (9465)	120	81	65	43	29	0
Paper 2 (9470)	120	94	69	58	35	0
Paper 3 (9475)	120	82	63	52	34	0

The cumulative percentage of candidates achieving each grade was as follows:

Paper	S	1	2	3	U
Paper 1 (9465)	6.5	17.1	45.1	69.7	100
Paper 2 (9470)	12.2	36.0	50.3	81.8	100
Paper 3 (9475)	13.0	38.0	56.9	82.1	100