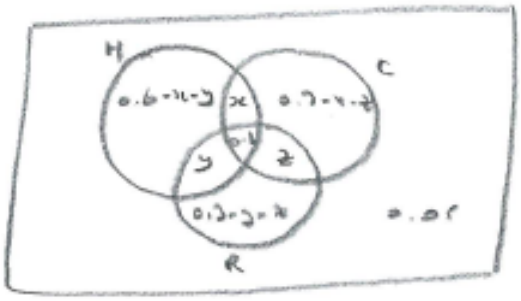
STEP 2005, Paper 1, Q12 – Solution (2 pages; 9/5/18)

In finding the range of possible values of q in (b), it is not easy to be sure that all of the constraints have been taken into account. The official sol'ns don't explain the reason behind the substitution $x = 0.6 - k$.

An alternative sol'n for this part is as follows:



[Note that Prob(Hat only) etc. have been expressed in terms of x, y & z – in order to keep the number of variables to a minimum.]

$q = \frac{x}{0.8}$

From the previous part, $x+y+z = 0.75$ (A).

[Although we are trying to find x, we can reduce the number of variables further by substituting $0.75 - y - z$ for x (which can be retrieved at the end).]

All the constraints can then be summarised by:

$0 \leq 0.6-x-y = 0.6 - (0.75-y-z) - y = z - 0.15$ (B)

$0 \leq 0.7-x-z = 0.7 - (0.75-y-z) - z = y - 0.05$ (C)

$0 \leq 0.3-y-z$ (D)

together with $x \geq 0$, $y \geq 0$ & $z \geq 0$

(B) & (C) then give:

$z \geq 0.15$ (which takes care of $z \geq 0$)

$y \geq 0.05$ (which takes care of $y \geq 0$)

This then gives $y+z \geq 0.05+0.15 = 0.2$

and (D) gives $y+z \leq 0.3$

Then, from (A), $0.75 - 0.3 \leq x \leq 0.75 - 0.2$; ie $0.45 \leq x \leq 0.55$,

so that $\frac{45}{80} \leq x \leq \frac{55}{80}$ or $\frac{9}{16} \leq x \leq \frac{11}{16}$