Logarithms (STEP) (2 pages; 14/7/21)

(1) $log_a b = c \Leftrightarrow a^c = b$

(2) eg $3 + 2log_2 5 = 3log_2 2 + log_2(5^2)$

$= log_2(2^3) + log_2(5^2) = log_2(8 \times 25) = log_2(200)$

(3) $log_a b \ log_b c = log_a c$ or $log_b c = \frac{log_a c}{log_a b}$

Proof: Let $b = a^x$ & $c = b^y$

Then $c = (a^x)^y = a^{xy}$

and $log_a c = xy = log_a b \ log_b c$

Special case: $log_b c = \frac{1}{log_c b}$

(4) As $log_8 8 = 1 \ and \ log_8 64 = 2$, and as $y = log_8 x$ is a concave function ($\frac{dy}{dx}$ is decreasing; ie $\frac{d^2y}{dx^2} < 0$), linear interpolation

$\Rightarrow log_8\left[\frac{1}{2}(8 + 64)\right] > \frac{1}{2}(1 + 2)$

ie $log_8 36 > \frac{3}{2}$

(5) To find an upper bound for eg $log_2 3$:

Suppose that $log_2 3 < \frac{m}{n}$

Then $3 < 2^{\left(\frac{m}{n}\right)}$ and $3^n < 2^m$

As $243 = 3^5 < 2^8 = 256$, $log_2 3 < \frac{8}{5}$

[and $\frac{8}{5}$ is a reasonably low upper bound, as 243 & 256 are reasonably close]

(6) eg $log_2 12 = log_2(3 \times 4) = log_2 3 + log_2 4 < \frac{8}{5} + 2 = \frac{18}{5}$,

from (5)

(7) eg $log_{36} 8 = \frac{1}{log_8 36} < \frac{2}{3}$, from (4)

(8) Example: Show that $log_5 10 < \frac{3}{2}$

$log_5 10 < \frac{3}{2} \Leftrightarrow 10 < 5^{\left(\frac{3}{2}\right)}$ (as the log function is increasing)

$\Leftrightarrow 10^2 < 5^3 \Leftrightarrow 100 < 125$