Polynomials – Q8 (26/6/23)

If the roots of the equation $x^3 + x^2 + x + 1 = 0$ are α, β & γ, find the equation with roots $\alpha + 1, \beta + 1$ & $\gamma + 1$

Solution

Method 1 (quicker)

Let $u = x + 1$.

Then $x = u - 1$, and $(u-1)^3 + (u-1)^2 + (u-1) + 1 = 0$,

so that $u^3 + u^2(-3+1) + u(3-2+1) + (-1+1-1+1) = 0$

and $u^3 - 2u^2 + 2u = 0$

Method 2 (good exercise)

Let the required eq'n be $u^3 + bu^2 + cu + d = 0$

Then $-b = (\alpha+1) + (\beta+1) + (\gamma+1)$

$= \alpha + \beta + \gamma + 3 = -1 + 3 = 2$

And $c = (\alpha+1)(\beta+1) + (\alpha+1)(\gamma+1) + (\beta+1)(\gamma+1)$

$= (\alpha\beta + \alpha\gamma + \beta\gamma) + (\alpha + \beta + \alpha + \gamma + \beta + \gamma) + 3$

$= 1 + 2(-1) + 3 = 2$

And $-d = (\alpha+1)(\beta+1)(\gamma+1)$

$= \alpha\beta\gamma + (\alpha\beta + \alpha\gamma + \beta\gamma) + (\alpha + \beta + \gamma) + 1$

$= (-1) + 1 + (-1) + 1 = 0$

So the required eq'n is $u^3 - 2u^2 + 2u = 0$