

Polynomials – Q6 (26/6/23)

If α , β and γ are the roots of the equation

$$x^3 - 14x^2 + 56x - 64 = 0,$$

find the equation with roots $\alpha\beta$, $\alpha\gamma$ and $\beta\gamma$.

Solution

$$\text{Let } u = \alpha\beta = \frac{\alpha\beta\gamma}{\gamma} = \frac{64}{\gamma}$$

Then $\gamma = \frac{64}{u}$ satisfies the original equation

Similarly for $u = \alpha\gamma$ and $u = \beta\gamma$.

Thus the required equation is

$$\left(\frac{64}{u}\right)^3 - 14\left(\frac{64}{u}\right)^2 + 56\left(\frac{64}{u}\right) - 64 = 0,$$

$$\text{giving } 4096 - 896u + 56u^2 - u^3 = 0$$

$$\text{or } u^3 - 56u^2 + 896u - 4096 = 0$$