

Polynomials – Q4 (26/6/23)

Find the roots of the equation $x^3 - 14x^2 + 56x - 64 = 0$,
given that they form a geometric progression.

Solution

Let the roots be $\frac{\alpha}{r}$, α & $r\alpha$

Then $\frac{\alpha}{r} \cdot \alpha \cdot r\alpha = 64$, so that $\alpha = 4$

Also $\frac{\alpha}{r} + \alpha + r\alpha = 14$, so that $\frac{1}{r} + 1 + r = \frac{7}{2}$

Then $2(1 + r + r^2) = 7r$ and $2r^2 - 5r + 2 = 0$

Hence $(2r - 1)(r - 2) = 0$ and so $r = \frac{1}{2}$ or 2

Thus the roots are 2, 4 and 8.