Polynomials - Q1 (26/6/23)

If the quadratic equation $2x^2 + 5x - 9 = 0$ has roots α and β, find the quadratic equation which has roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$

Solution

Method 1

$\alpha + \beta = -\frac{5}{2}$ and $\alpha\beta = -\frac{9}{2}$

Let the new equation be $x^2 + bx + c = 0$

Then $\frac{1}{\alpha} + \frac{1}{\beta} = -b$ and $\frac{1}{\alpha} . \frac{1}{\beta} = c$,

so that $b = \frac{-(\alpha+\beta)}{\alpha\beta} = -\frac{5}{9}$ and $c = -\frac{2}{9}$

and the new equation is $x^2 - \frac{5x}{9} - \frac{2}{9} = 0$

or $9x^2 - 5x - 2 = 0$

[Note that, if written as $-9x^2 + 5x + 2 = 0$, then the coefficients of the original equation are reversed.]

Method 2

Let $u = \frac{1}{x}$, so that $x = \frac{1}{u}$

Then $2\left(\frac{1}{u}\right)^2 + \frac{5}{u} - 9 = 0$

and $2 + 5u - 9u^2 = 0$ or $9u^2 - 5u - 2 = 0$