Polar Curves - Summary (11 pages; 2/11/20)

[See "Polar Curves" for further details.]

Contents

(1) Checklist for sketching graphs

(i) Plot points for convenient values of θ, such as 0 & $\frac{\pi}{2}$.

(ii) Any function of $cos\theta$ will be symmetric about the x axis (as $\cos(-\theta) = cos\theta$).

(iii) Any function of $sin\theta$ will be symmetric about the y axis (as

$\sin(\pi - \theta) = sin\theta$).

(iv) The value(s) of θ for which $r = 0$ will determine the direction(s) from which the curve approaches the Origin.

(2) Notes on sketching graphs

(i) The points $(1, 1)$ & $(-1, -1)$ have the same value of $tan\theta$ (since $\tan(\frac{5\pi}{4}) = tan\left(\frac{\pi}{4}\right)$). We therefore have to consider which quadrant the point is in.

(ii) To help with drawing the parts of graphs where $r < 0$ (usually shown as a dotted curve - but sometimes omitted), imagine the hand of a clock sweeping round, but in an anti-clockwise direction (to keep track of θ).

(iii) $1 + cos0 = 1 + sin\left(\frac{\pi}{2}\right)$, so that $\theta = 0$ for $r = 1 + cos\theta$ (for example) corresponds to $\theta = \frac{\pi}{2}$ for $\mathrm{r} = 1 + sin\theta$, and so the graph of $r = 1 + sin\theta$ can be obtained from that of $\mathrm{r} = 1 + \cos\theta$ by a rotation of $\frac{\pi}{2}$ anti-clockwise.

(iv) $r = f(sin[n\theta])$ can be obtained from $r = f(cos[n\theta])$ by an anti-clockwise rotation of $\frac{(\frac{\pi}{2})}{n}$ [As $r = f(cos[n\theta])$ is symmetric about the x-axis, whereas $r = f(sin[n\theta])$ is only symmetric about $\theta = \frac{(\frac{\pi}{2})}{n}$]

(v) Where there are parts of the curve for which $r < 0$, there may be apparent symmetry. For example, it appears that $r = \sin(2\theta)$ is symmetric about the y-axis, but the apparent reflection of $r = \sin(2\alpha)$ in the y-axis, where $0 < \alpha < \frac{\pi}{2}$, in fact arises from $r = \sin(2[-\alpha])$, for which $r < 0$.

(3) $\boldsymbol{r = cos(n\theta)}$ & $\boldsymbol{r = sin(n\theta)}$ families

Graphs of the form $r = cos(n\theta)$ and $r = sin(n\theta)$ consist of n petals with $r = 1$ at the end of the petal, together with n petals with $r = -1$ at the end of the petal.

When n is odd, the ' $r = -1$' petals overlap with the ' $r = 1$' petals, giving a total of n visible petals.

When n is even, there is no overlapping, and so there are $2n$ visible petals.

(4) $\boldsymbol{r = \lambda + cos\theta}$ family

(Sometimes represented in the form $r = a(p + qcos\theta)$, in which case $\frac{p}{q}$ has the role of λ. The presence of the a doesn't affect the shape of the curve.)

The values of λ can be classified as follows:

$\lambda = 0$: circle

$0 < \lambda < 1$

$\lambda = 1$: 'cardioid'

$1 < \lambda < 2$: 'dimple'

$\lambda \geq 2$: 'egg'

(See Appendix for examples of these.)

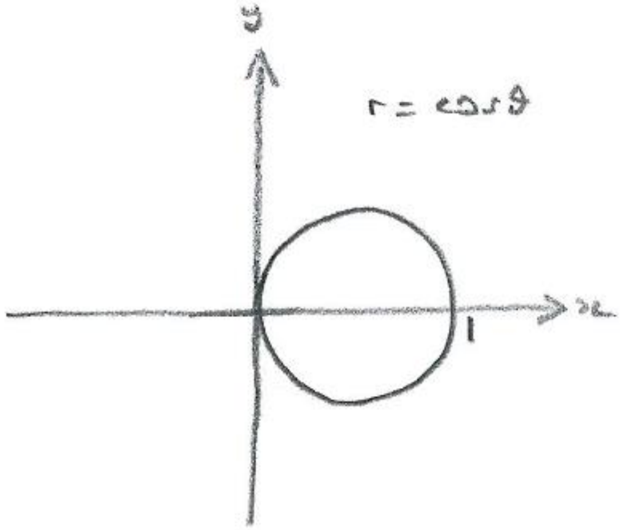
(5) $x = a$ and $y = b$ in polar form

$x = a$ & $x = rcos\theta \Rightarrow rcos\theta = a \Rightarrow \mathrm{r} = asec\theta$

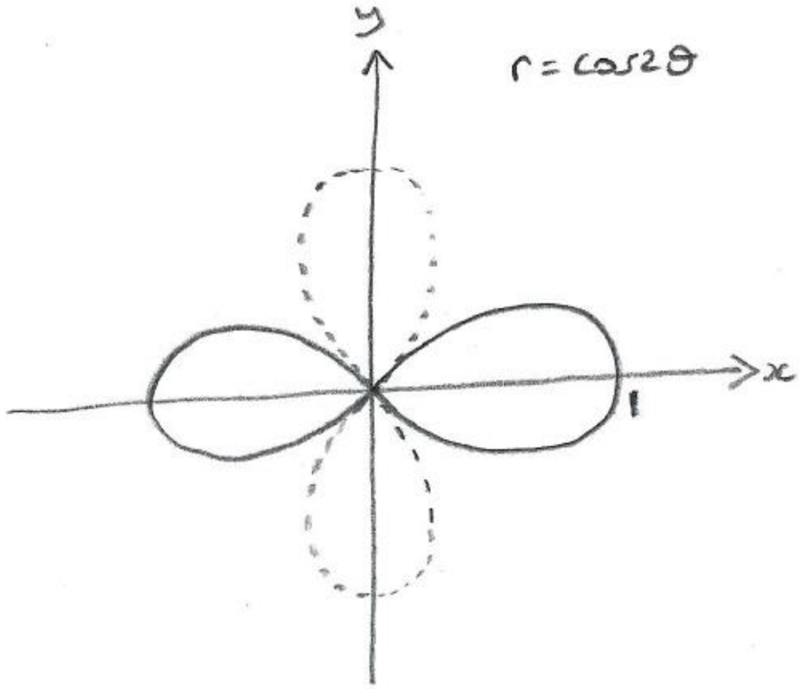
$y = b$ & $y = rsin\theta \Rightarrow rsin\theta = b \Rightarrow r = bcosec\theta$

Appendix: Example curves

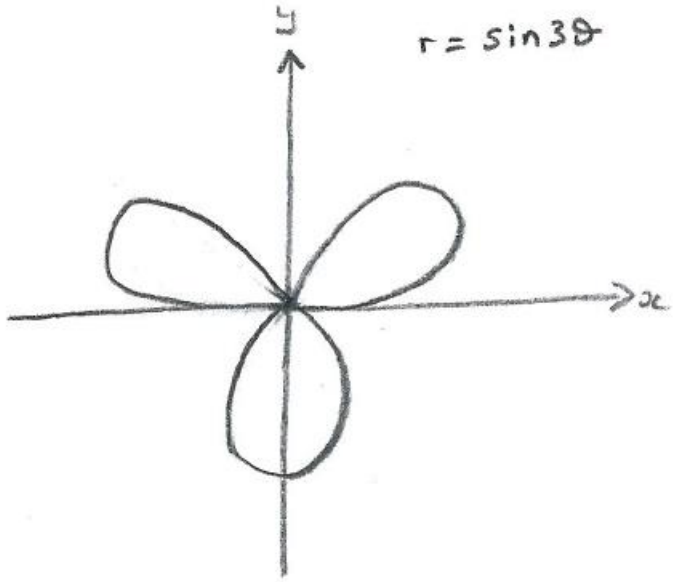
(a) $r = \cos\theta$ (circle)



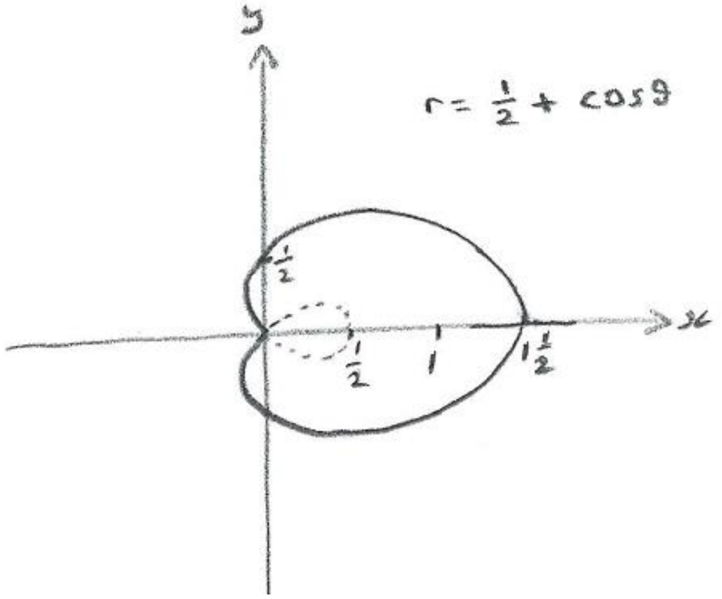
(b) $r = \cos(2\theta)$



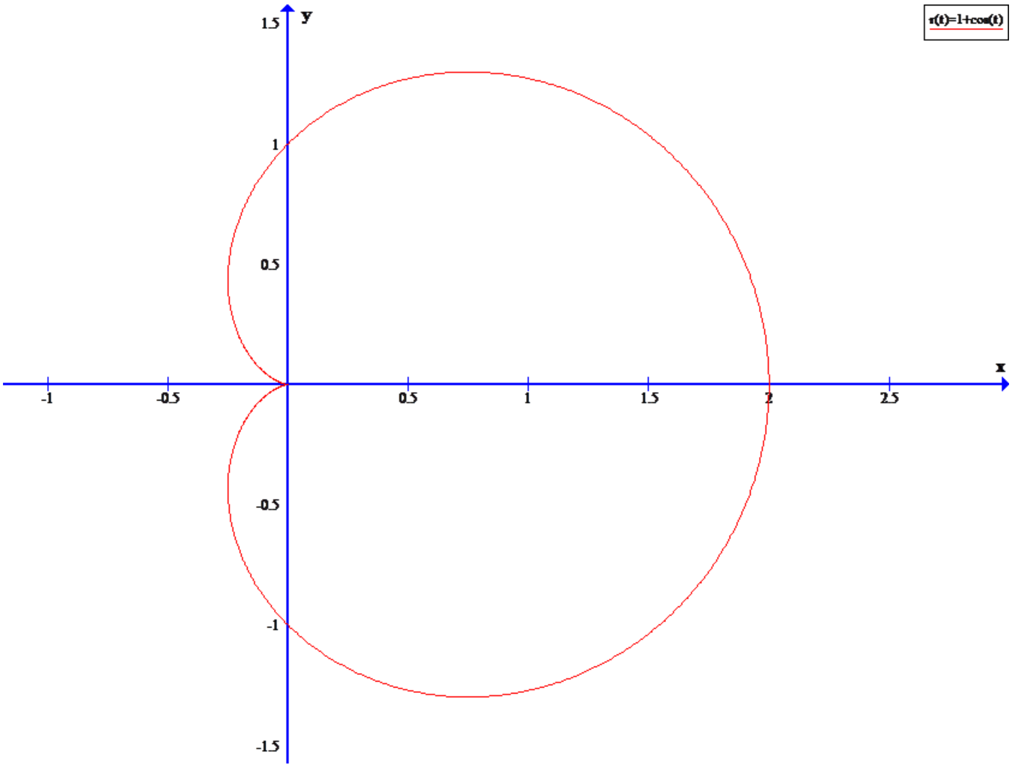
(c) $r = sin(3\theta)$



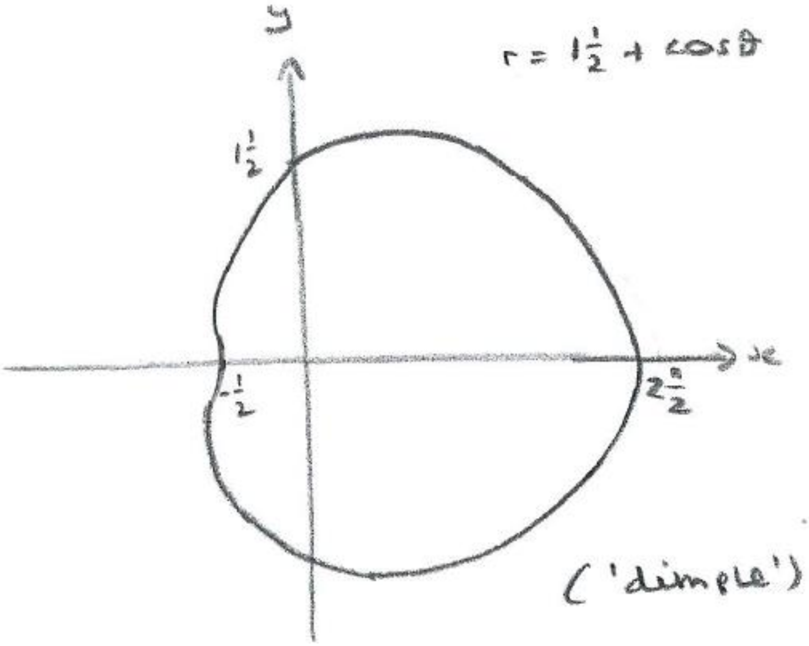
(d) $r = \frac{1}{2} + cos\theta$



(e) $r = 1 + \cos\theta$ ('cardioid')



(f) $r = \frac{3}{2} + cos\theta$ ('dimple')



(g) $r = 2 + cos\theta$ ('egg')