Polar Curves – Q5 [Practice/M](16/6/23)

A curve has polar equation $r = 3(cos\theta + 2sin\theta)$, for $0 \leq \theta \leq \pi$.

(i) Show that the curve is a circle.

(ii) Determine the polar coordinates of the point on the curve which is furthest from the pole.

Solution

(i) $r = 3(cos\theta + 2sin\theta) \Rightarrow r^2 = 3rcos\theta + 6rsin\theta,$

$\Rightarrow\; x^2 + y^2 = 3x + 6y$

$\Rightarrow\; (x - \frac{3}{2})^2 + (y - 3)^2 - \frac{3^2}{4} - 3^2 = 0$

$\Rightarrow\; (x - \frac{3}{2})^2 + (y - 3)^2 = \frac{5(9)}{4}$; ie a circle with radius $\frac{3\sqrt{5}}{2}$

(ii) $\frac{dr}{d\theta} = 3(-sin\theta + 2cos\theta)$

$\frac{dr}{d\theta} = 0 \Rightarrow tan\theta = 2$ (as $cos\theta \neq 0$)

$\frac{d^2r}{d\theta^2} = 3(-cos\theta - 2sin\theta) = -3cos\theta(1 + 2tan\theta)$ (when $cos\theta \neq 0$)

So when $tan\theta = 2, \frac{d^2r}{d\theta^2} < 0$, as $0 < \theta < \frac{\pi}{2}$, so that $cos\theta > 0$,

and hence r is a maximum when $tan\theta = 2$

When $tan\theta = 2, sec^2\theta = 2^2 + 1 = 5$,

so that $cos\theta = \sqrt{\frac{1}{5}}\ (as\ cos\theta > 0)$

and $r = 3cos\theta(1 + 2tan\theta) = \frac{3}{\sqrt{5}}(1 + 4) = 3\sqrt{5}$

The polar coordinates are thus $(3\sqrt{5}\,,\ tan^{-1}2)$.