Partial Fractions - Q1 [Practice/M] (30/5/21)

Express $\frac{1}{(1-x^2)^2}$ in terms of partial fractions

Express $\frac{1}{(1-x^2)^2}$ in terms of partial fractions

Solution

$$\frac{1}{(1-x^2)^2} = \frac{1}{(1-x)^2(1+x)^2}$$

$$= \frac{A}{1-x} + \frac{B}{(1-x)^2} + \frac{C}{1+x} + \frac{D}{(1+x)^2}$$

so that $1 = A(1-x)(1+x)^2 + B(1+x)^2 + C(1+x)(1-x)^2 + D(1-x)^2$

Then $x = 1 \Rightarrow 1 = 4B \Rightarrow B = \frac{1}{4}$

$x = -1 \Rightarrow 1 = 4D \Rightarrow D = \frac{1}{4}$

$x = 0 \Rightarrow 1 = A + B + C + D \Rightarrow A + C = \frac{1}{2}$

Equating coefficients of $x^3 \Rightarrow 0 = -A + C$

Hence $A = C = \frac{1}{4}$ and $\frac{1}{(1-x^2)^2} = \frac{1}{4(1-x)} + \frac{1}{4(1-x)^2} + \frac{1}{4(1+x)} + \frac{1}{4(1+x)^2}$