

Parabolas Q4 [5 marks] (30/5/21)

Exam Boards

OCR : -

MEI: -

AQA: -

Edx: Further Pure 1 (Year 1)

If the tangents to a parabola at P and Q are perpendicular, show that the chord PQ passes through the focus S of the parabola.

[The equation of the tangent can be used without proof.]

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Solution

The gradients of the two tangents are $\frac{1}{p}$ and $\frac{1}{q}$ (standard result).

[1 mark]

As the tangents are perpendicular, $\left(\frac{1}{p}\right)\left(\frac{1}{q}\right) = -1$, so that $pq = -1$.

[1 mark]

Gradient of PS = $\frac{2ap-0}{ap^2-a} = \frac{2p}{p^2-1}$, [1 mark]

and the gradient of

$$QS = \frac{2aq-0}{aq^2-a} = \frac{2q}{q^2-1}$$

We wish to show that these gradients are the same; ie that

$$\frac{2p}{p^2-1} = \frac{2q}{q^2-1} \text{ [1 mark]}$$

$$\text{LHS} = \frac{2\left(-\frac{1}{q}\right)}{\left(-\frac{1}{q}\right)^2-1} = \frac{2q}{-1+q^2} = \text{RHS} \text{ [1 mark]}$$