Parabolas Q1 [4 marks] (29/5/21)

Exam Boards

OCR : -

MEI: -

AQA: -

Edx: Further Pure 1 (Year 1)

Using the parametric equations of a parabola $(x = at^2, y = 2at)$, show that the midpoints of chords of a parabola that have the same direction lie on a straight line parallel to the x-axis.

[A chord of a parabola joins two points on the parabola.]

[4 marks]

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Solution

Let points P and Q on the parabola have parameters t_1 & t_2.

The chord PQ has gradient $\frac{2at_2 - 2at_1}{at_2{}^2 - at_1{}^2} = \frac{2(t_2 - t_1)}{t_2{}^2 - t_1{}^2} = \frac{2}{t_1 + t_2}$, and we are told that this is constant. [2 marks]

The y-coordinate of the midpoint of PQ is $\frac{1}{2}(2at_1 + 2at_2) = a(t_1 + t_2)$, [1 mark]

which is constant, as $\frac{2}{t_1 + t_2}$ and therefore $t_1 + t_2$ are constant, giving the required result. [1 mark]