Probability Generating Function Q2 [Problem/H]
(12/6/21)

A hen lays N eggs, where $N \sim P_o(\lambda)$, and each egg has probability p of hatching. Using any results about probability generating functions , show that the total number of eggs that hatch $\sim P_o(\lambda p)$

['Poisson hen']

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Solution

Let the total number of eggs that hatch be $Z = X_1 + \cdots + X_N$,

where the $X_i \sim \text{Bernouilli}(p)$.

Then $G_Z(s) = G_N\big(G_X(s)\big)$

with $G_N(s) = e^{\lambda(s-1)}$ and $G_X(s) = (1-p) + ps$,

so that $G_Z(s) = e^{\lambda(-p+ps)} = e^{\lambda p(s-1)}$

and hence $Z \sim P_o(\lambda p)$, as required.