Oscillations – Q4 [14 marks] (20/6/21)

Exam Boards

OCR : Mechanics (Year 2)

MEI: Mechanics b

AQA: Mechanics (Year 1)

Edx: Mechanics 1 (Year 2)

Two light springs are attached to a sphere S of mass 2kg that rests on a smooth horizontal surface. The other ends of the springs are attached to fixed points A and B, $3m$ apart (the sphere has negligible dimensions). Spring AS has stiffness 20 Nm^{-1} and spring SB has stiffness 30 Nm^{-1}. Both springs have a natural length of $1m$. The sphere oscillates along the line AB and has a maximum speed of $2ms^{-1}$.

(i) If x is the distance of the sphere from A, show that

$\frac{d^2x}{dt^2} = 40 - 25x$ [4 marks]

(ii) How far is the sphere from A when it is moving at its maximum speed? [2 marks]

(iii) What is the closest that the sphere gets to A? [8 marks]

Solution

(i) By Hooke's law, and N2L: $30([3-x]-1)-20(x-1)=2\frac{d^2x}{dt^2}$

[Hooke's law: 2 marks; N2L: 1 mark]

so that $\frac{d^2x}{dt^2}=\frac{1}{2}(80-50x)=40-25x$ [1 mark]

(ii) The maximum speed occurs when $\frac{d^2x}{dt^2}=0$ [1 mark]

so that $40-25x=0$ and $x=\frac{8}{5}=1.6m$ [1 mark]

(iii) When the sphere has its maximum speed of $2ms^{-1}$,

KE $=\frac{1}{2}(2)2^2=4\,J$ [1 mark]

and its elastic PE $=\frac{1}{2}(20)(1.6-1)^2+\frac{1}{2}(30)([3-1.6]-1)^2$

$=3.6+2.4=6\,J$ [2 marks]

At its closest approach to A, $KE=0$

and EPE $=\frac{1}{2}(20)(x-1)^2+\frac{1}{2}(30)([3-x]-1)^2$ [2 marks]

$=10(x^2-2x+1)+15(4-4x+x^2)$

$=25x^2-80x+70\ J$ [1 mark]

By conservation of energy, $25x^2-80x+70=4+6$

$\Rightarrow 5x^2-16x+12=0$ [1 mark]

$\Rightarrow x=\frac{16\pm\sqrt{256-240}}{10}=1.6\pm0.4$

[1.6 is the centre of the oscillations, and agrees with (ii)]

So the sphere's closest distance is $1.2m$ [1 mark]