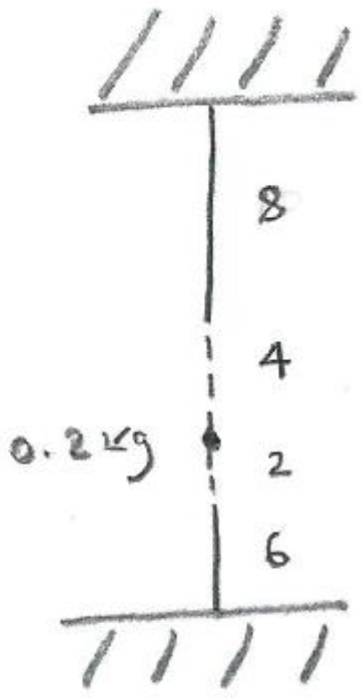
Oscillations – Q3 [Problem/H](16/6/21)

A 0.2 kg mass is held between two elastic strings, as shown in the diagram. The upper string has original length 8m and modulus of elasticity 2N, and is initially extended by 4m. The lower string has original length 6m and modulus of elasticity 1N, and is initially extended by 2m. When the mass is released, determine its subsequent motion (assume $g = 10$, and ignore any resistance to motion).



Solution

Let x be the displacement of the mass above its initial position.

By N2L, $(0.2)\ddot{x} = \frac{4-x}{8}(2) - \frac{2+x}{6}(1) - (0.2)(10)$

so that $\ddot{x} = 5\left(1 - \frac{1}{3} - 2\right) - 5x\left(\frac{1}{4} + \frac{1}{6}\right) = -\frac{20}{3} - \frac{25x}{12}$

Method 1

Writing $\ddot{x} + \frac{25x}{12} = -\frac{20}{3}$ gives a complementary function of

$x = Acos(\frac{5}{\sqrt{12}}t + \alpha)$

and a trial function of $x = C$ for the particular integral gives

$\frac{25C}{12} = -\frac{20}{3}$, so that $C = -\frac{16}{5}$

and the general solution is $x = Acos\left(\frac{5}{\sqrt{12}}t + \alpha\right) - \frac{16}{5}$

so that $\dot{x} = -\frac{5}{\sqrt{12}}Asin\left(\frac{5}{\sqrt{12}}t + \alpha\right)$

Then $t = 0, \dot{x} = 0 \Rightarrow \alpha = 0$

and $t = 0, x = 0 \Rightarrow 0 = A - \frac{16}{5}$

so that the particular solution is $x = \frac{16}{5}[cos\left(\frac{5}{\sqrt{12}}t\right) - 1]$

This is SHM of amplitude $\frac{16}{5}$ about $x = -\frac{16}{5}$

The period of oscillations is given by $\frac{5}{\sqrt{12}}T = 2\pi$,

and is $T = \frac{2\pi\sqrt{12}}{5} = 4.35$ s (3sf)

Thus the mass falls when released, and rises again to its initial position (as would be predicted by conservation of energy).

Method 2

$\ddot{x} = -\frac{20}{3} - \frac{25x}{12}$

Let $-\frac{20}{3} - \frac{25x}{12} = -\frac{25y}{12}$, so that $y = x + \frac{20}{3}\left(\frac{12}{25}\right) = x + \frac{16}{5}$

Then $\ddot{y} = \ddot{x} = -\frac{25y}{12}$

which gives SHM of amplitude $\frac{16}{5}$ about $y = 0$; ie $x = -\frac{16}{5}$