Oscillations – Q2 [Problem/H](16/6/21)

A flexible bar is embedded horizontally in a wall. A particle rests on the free end of the bar, and the bar (with the particle) is pulled down 2cm below the horizontal, and then released. Given that the bar and particle start to perform simple harmonic motion about the horizontal position, with 5 cycles per second, how long is it before the particle loses contact with the bar, and what speed does it have at that point? [Note: The particle will not be in contact with the bar long enough to complete a cycle of the simple harmonic motion.]

Solution

Let x be the displacement of the bar from the horizontal, where upwards is the positive direction.

Then (converting to S.I. units), $x = 0.02sin(\omega t)$, if (for convenience) we measure time from when $x = 0$.

Then, as 1 cycle (ie 2π radians) takes $\frac{1}{5}$ sec.,

$\omega\left(\frac{1}{5}\right) = 2\pi$, so that $\omega = 10\pi$

and $\ddot{x} = -\omega^2 x$

The particle is subject to gravity and a reaction force $R(x)$ from the bar, so that

$R(x) - mg = m\ddot{x}$, whilst the particle is in contact with the bar

(where m is the mass of the particle)

The particle loses contact with the bar when $R(x) = 0$;

ie when $-mg = m\ddot{x}$ and $\ddot{x} = -g$

As $\ddot{x} = -\omega^2 x$, this occurs when $-\omega^2 x = -g$; ie $x = \frac{g}{\omega^2}$

(note that this is where $x > 0$; ie above the horizontal).

The bar and particle take a quarter of a cycle, ie $\frac{1}{20}$ sec., to travel from the point of release to the horizontal.

If contact is lost at T secs after the horizontal has been reached,

$\frac{g}{\omega^2} = x = 0.02sin(\omega T)$, where $\omega = 10\pi$.

Then $T = \frac{1}{10\pi}sin^{-1}\left(\frac{9.8}{100\pi^2(0.02)}\right) = 0.016537$

and hence the required time from release is $\frac{1}{20} + 0.016537 =$ $0.066537 = 0.0665$ s (3sf)

The speed of the particle is $\dot{x}(T) = 0.02(10\pi)\cos{(10\pi T)}$

$= 0.54542\ ms^{-1}$ or $54.5cms^{-1}$(3sf)