Oscillations – Q1 [Problem/H](16/6/21)

A lift has an elastic string suspended from its ceiling, with a mass of 10 grams at the end of the string. The string has natural length 80 cm, and modulus of elasticity $20N$. Initially, when the lift is stationary, the mass is hanging in equilibrium. The lift then starts to ascend with an acceleration of 0.2 ms^{-2}. Show that the extension of the string after t $secs$ is $0.4 - 0.008\cos(50t)$ cm.

[Assume that $g = 9.8ms^{-2}$]

Solution

Let x be the distance of the mass below the level of the ceiling of the lift when it is stationary, measured relative to the lift's surroundings.

Then $x = 0.8 + e - y$,

where e is the extension of the string and y is the distance moved (upwards) by the lift,

so that $\ddot{x} = \ddot{e} - \ddot{y} = \ddot{e} - 0.2$

Considering the forces on the mass,

$0.01g - T = 0.01\ddot{x}$,

where T is the tension in the string,

and by Hooke's law, $T = \frac{20}{0.8}e$

So $0.01g - \frac{20}{0.8}e = 0.01\ddot{x} = 0.01(\ddot{e} - 0.2)$,

and hence $g - 2500e = \ddot{e} - 0.2$,

or $\ddot{e} + 2500e = 9.8 + 0.2 = 10$ (*)

To solve the differential equation:

the auxiliary equation is $\lambda^2 + 2500 = 0$,

with roots $\lambda = \pm 50i$,

so that the complementary function is $Ae^{50it} + Be^{-50it}$

or $(A + B)cos50t + (A - B)isin50t$

or $Ccos50t + Dsin50t$,

which can be written as $Ecos(50t + \alpha)$

The particular integral of the differential equation is a constant F (as the RHS of (*) is a constant),

such that $2500F = 10$, so that $F = 0.004$

Thus the general solution of (*) is:

$e = Ecos(50t + \alpha) + 0.004$ (**)

and $\dot{e} = -50Esin(50t + \alpha)$

When $t = 0$, and the mass is hanging in equilibrium,

$0.01g - T = 0$ and $T = \frac{20}{0.8}e$,

so that $0.01g = \frac{20}{0.8}e$ and $e = \frac{49}{12500}$

Also, at $t = 0$, $\dot{e} = 0$, so that $\alpha = 0$

Thus, from (**), $\frac{49}{12500} = E + 0.004$,

and $E = -0.00008$,

so that $e = 0.004 - 0.00008cos(50t)\ m$

or $0.4 - 0.008cos(50t)\ cm$