

# Numerical Solution of Equations - Comparison of methods

(2 pages; 22/10/18)

## (1) Convergence

Provided the function is continuous, the Bisection method and method of False Position will always lead to a root (excluding the case of repeated roots).

The Fixed-point, Newton-Raphson and Secant methods will not converge to a root in all cases.

Typical order of speed of convergence (starting with the fastest):

Newton-Raphson

Fixed Point (with relaxed iteration)

Secant

Fixed Point (without relaxed iteration)

False Position

Bisection

## (2) Problems

The Bisection method may not detect a further root within an interval. It doesn't cope with repeated roots.

The Newton-Raphson method may not work if  $x_0$  is close to a stationary point. Also  $x_1$  may be outside the domain of the function, or it may lead to a different root (eg one that has already been found).

### (3) Ease of use

For the Fixed-point method, a suitable rearrangement has to be found - although relaxed iteration may be employed.

The Bisection method and the method of False Position automatically provide an interval estimate for the root. The Fixed-point method provides an interval estimate  $(x_{n-1}, x_n)$ , or the other way round, if  $g'(\alpha) < 0$ . If  $g'(\alpha) > 0$ , and for the Newton-Raphson and Secant methods, an interval has to be found by looking for a change of sign at neighbouring points.

The Bisection method and the method of False Position are less easily automated than the Fixed-point, Newton-Raphson and Secant methods (as a new interval has to be determined at each stage, with an examination for change of sign). The Newton-Raphson and Fixed Point methods can be applied quickly with a calculator.

For the Bisection method, the number of dps can become unwieldy.